

Succession of LaRouche-Riemann Manifolds

$n + 2$



$n+1$



n



Gottfried Leibniz (1646-1716)

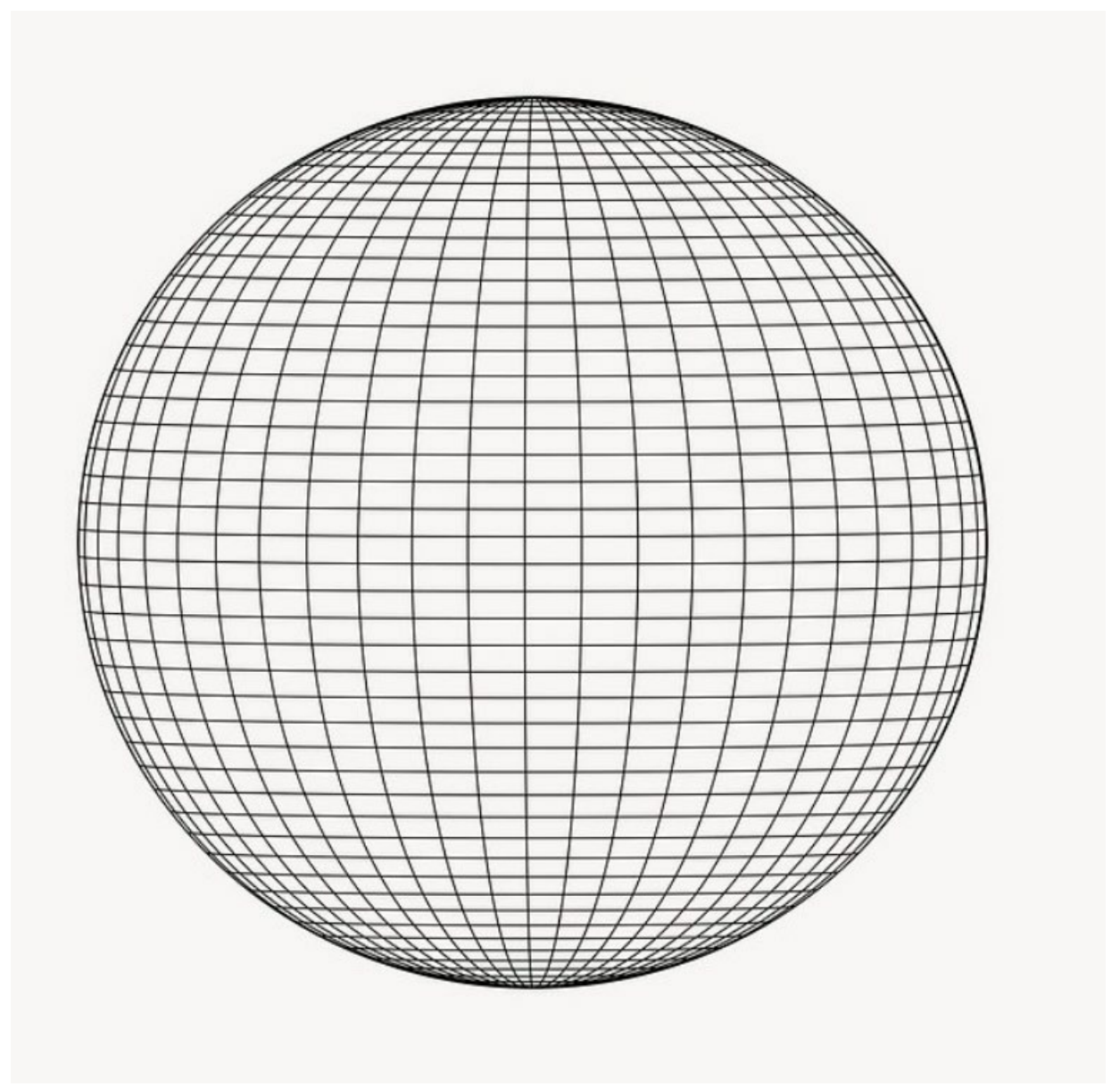
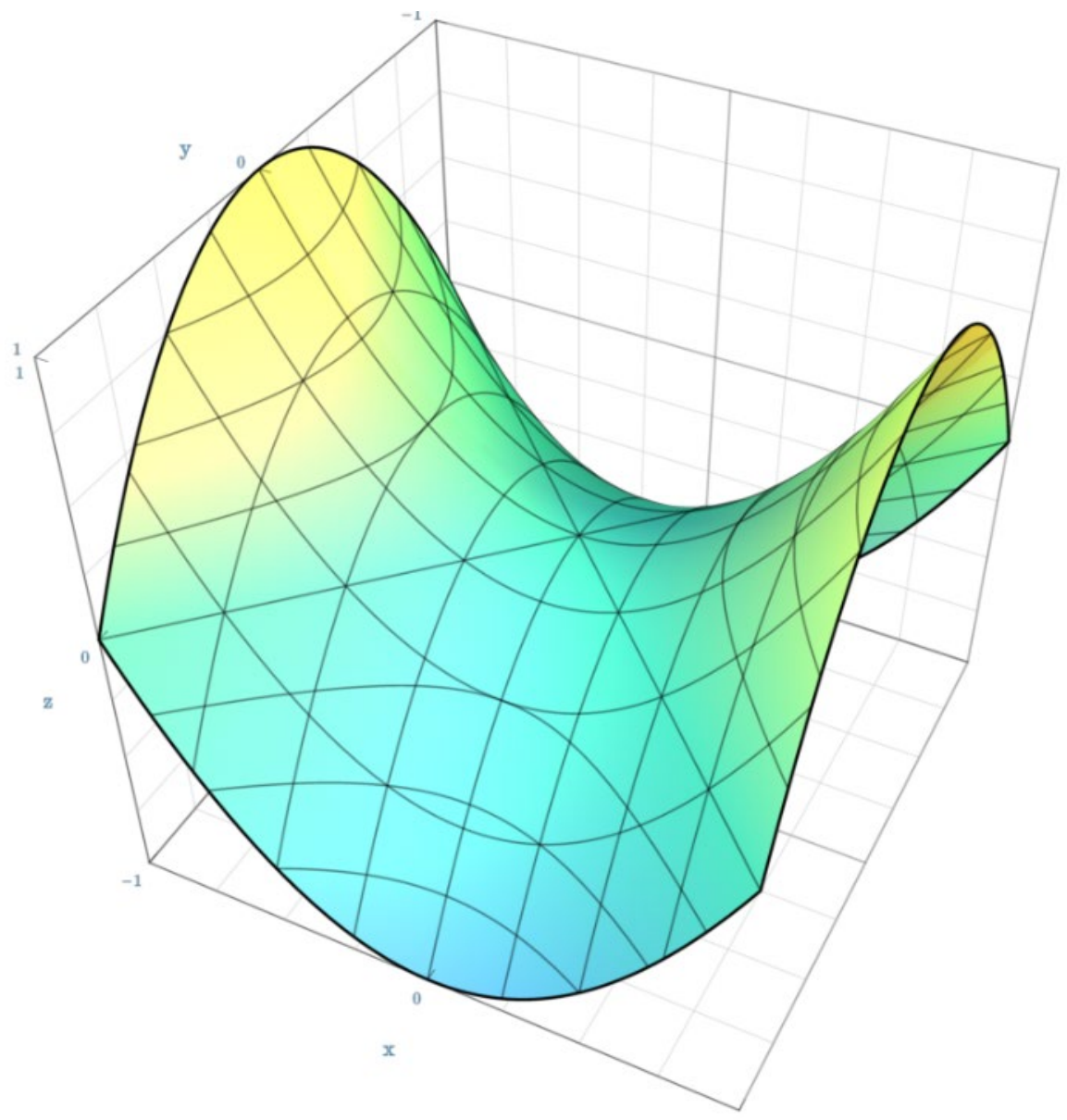


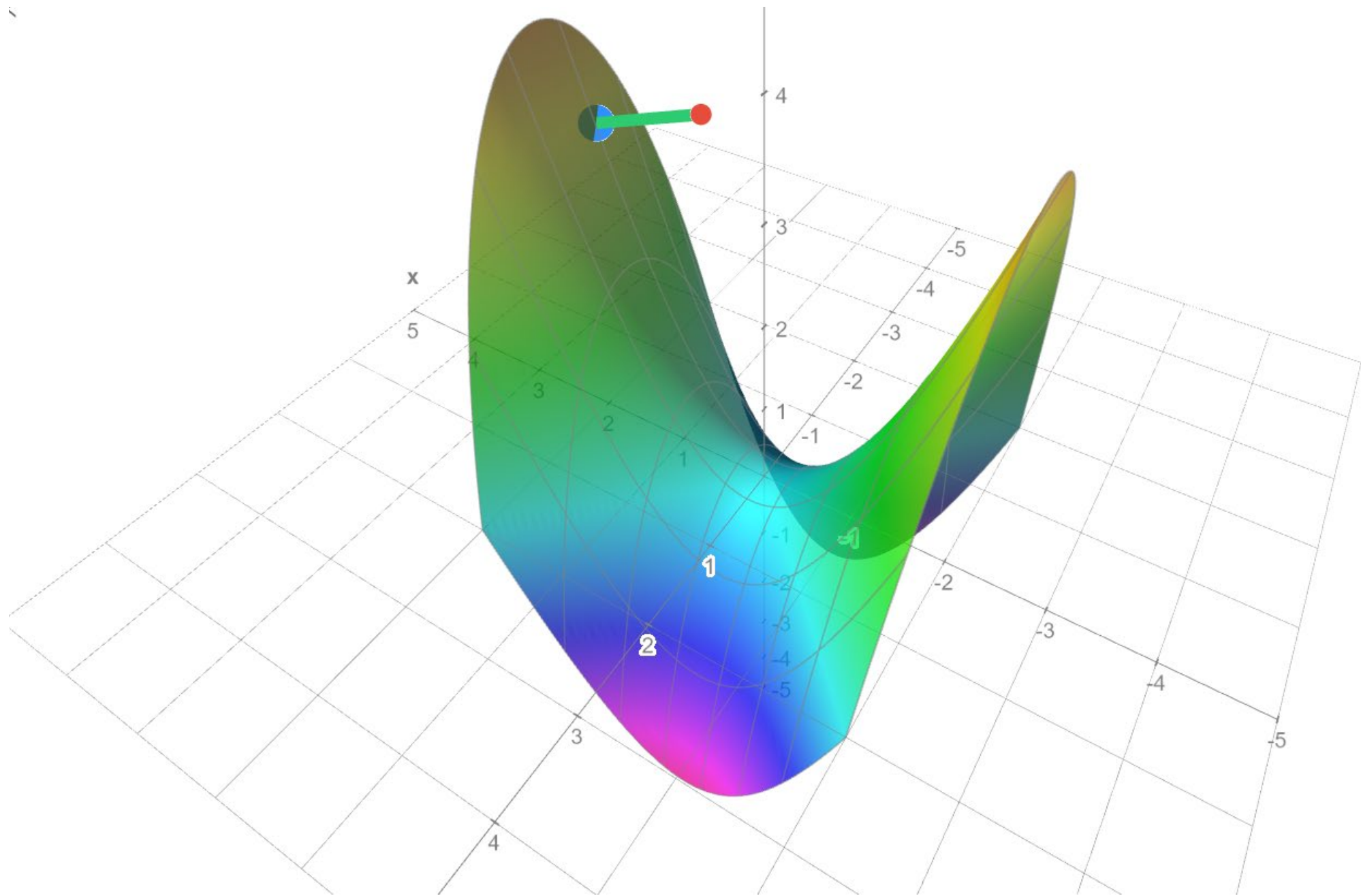
Portrait by Christian Albrecht Jensen

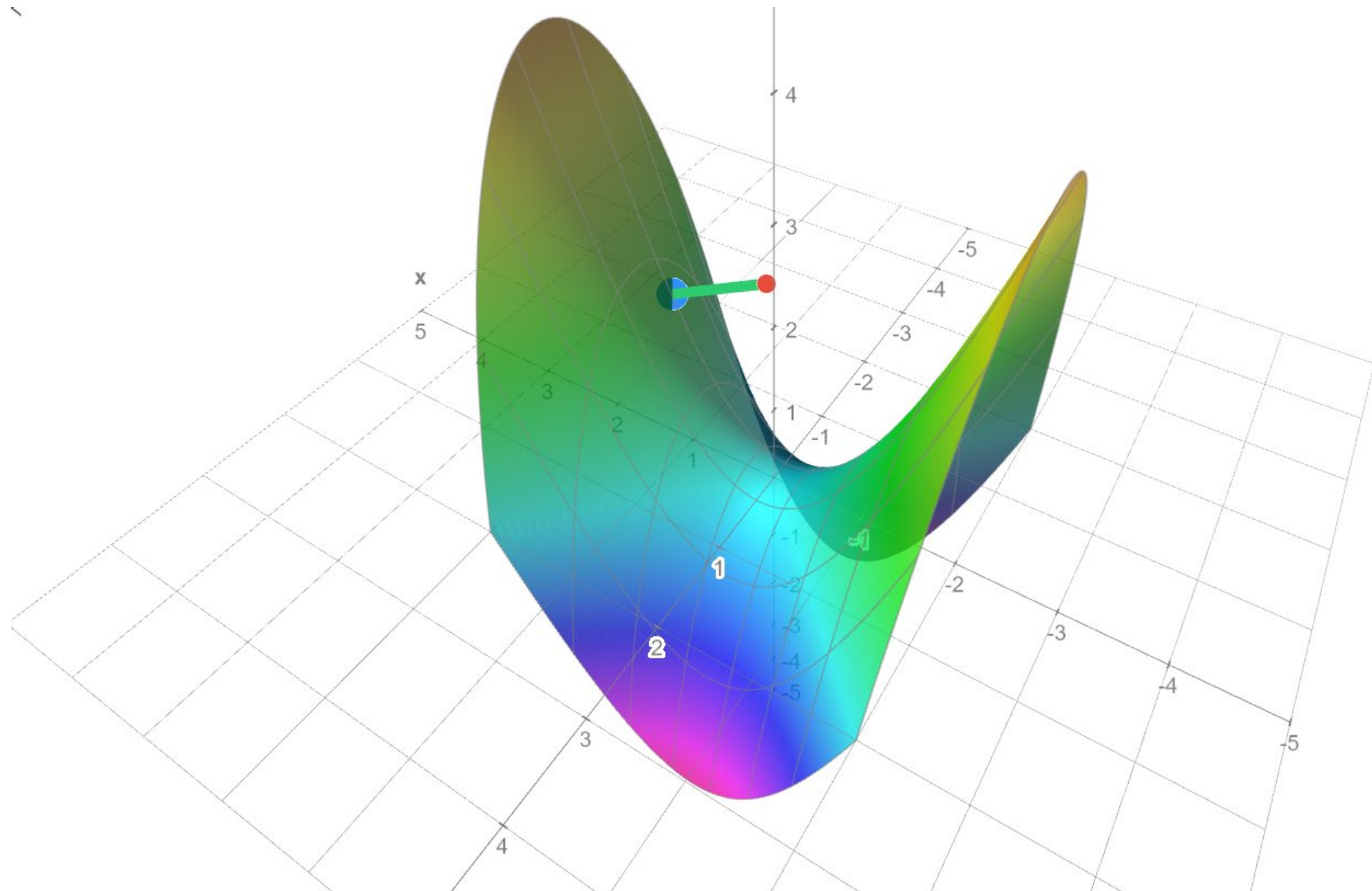
Carl Friedrich Gauss (1777-1855)



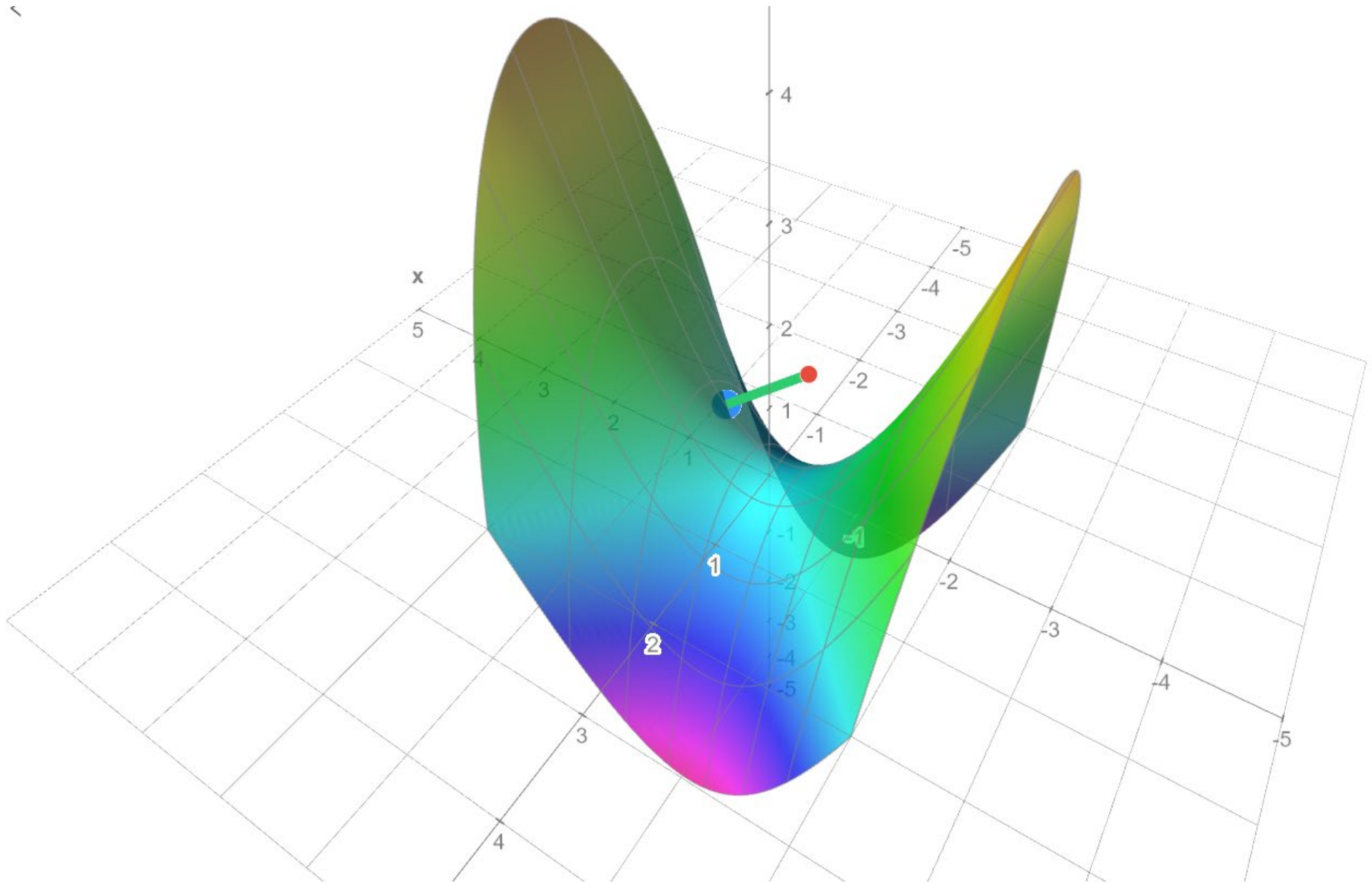
Bernhard Riemann (1826-1866)



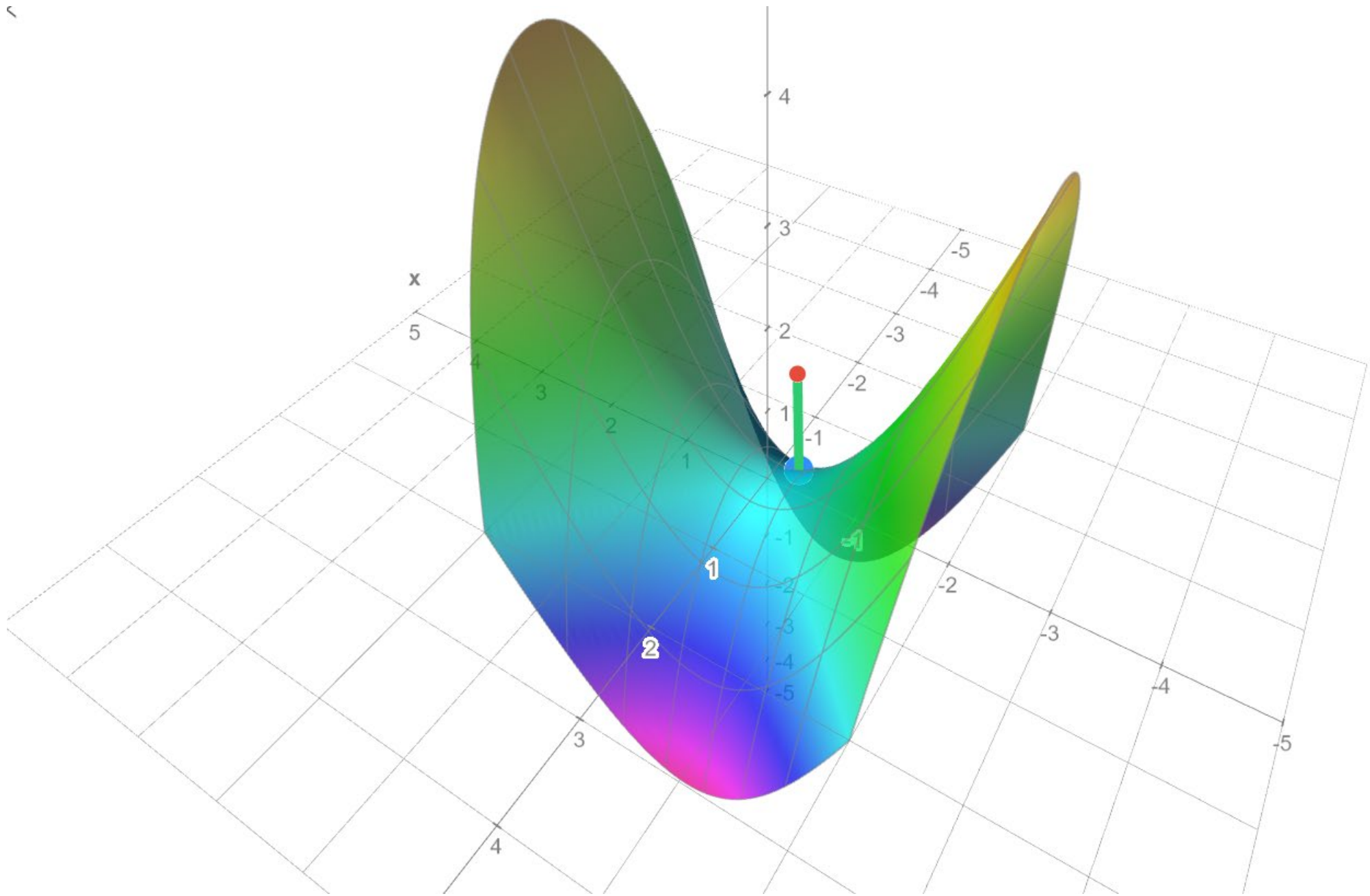


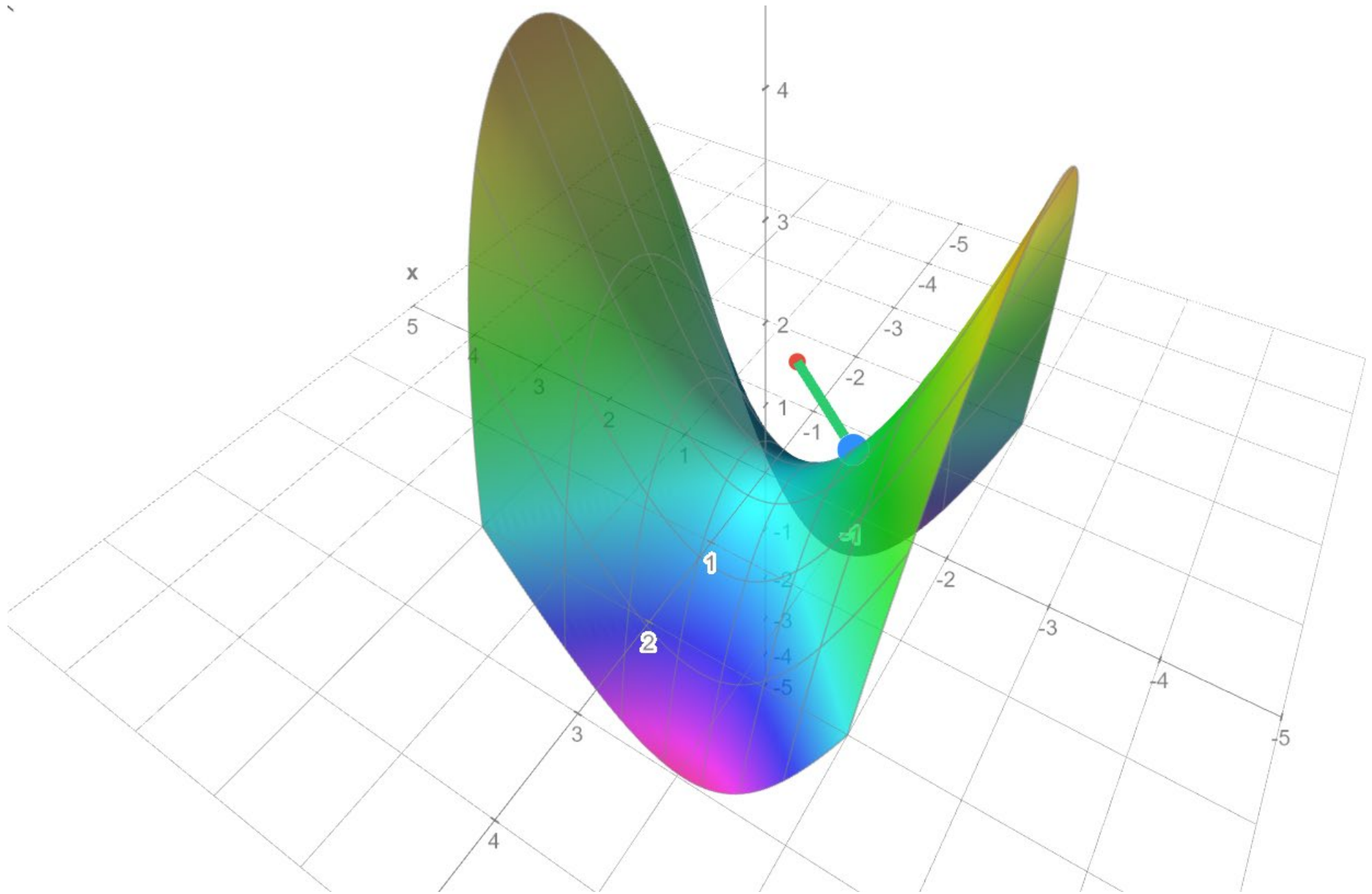


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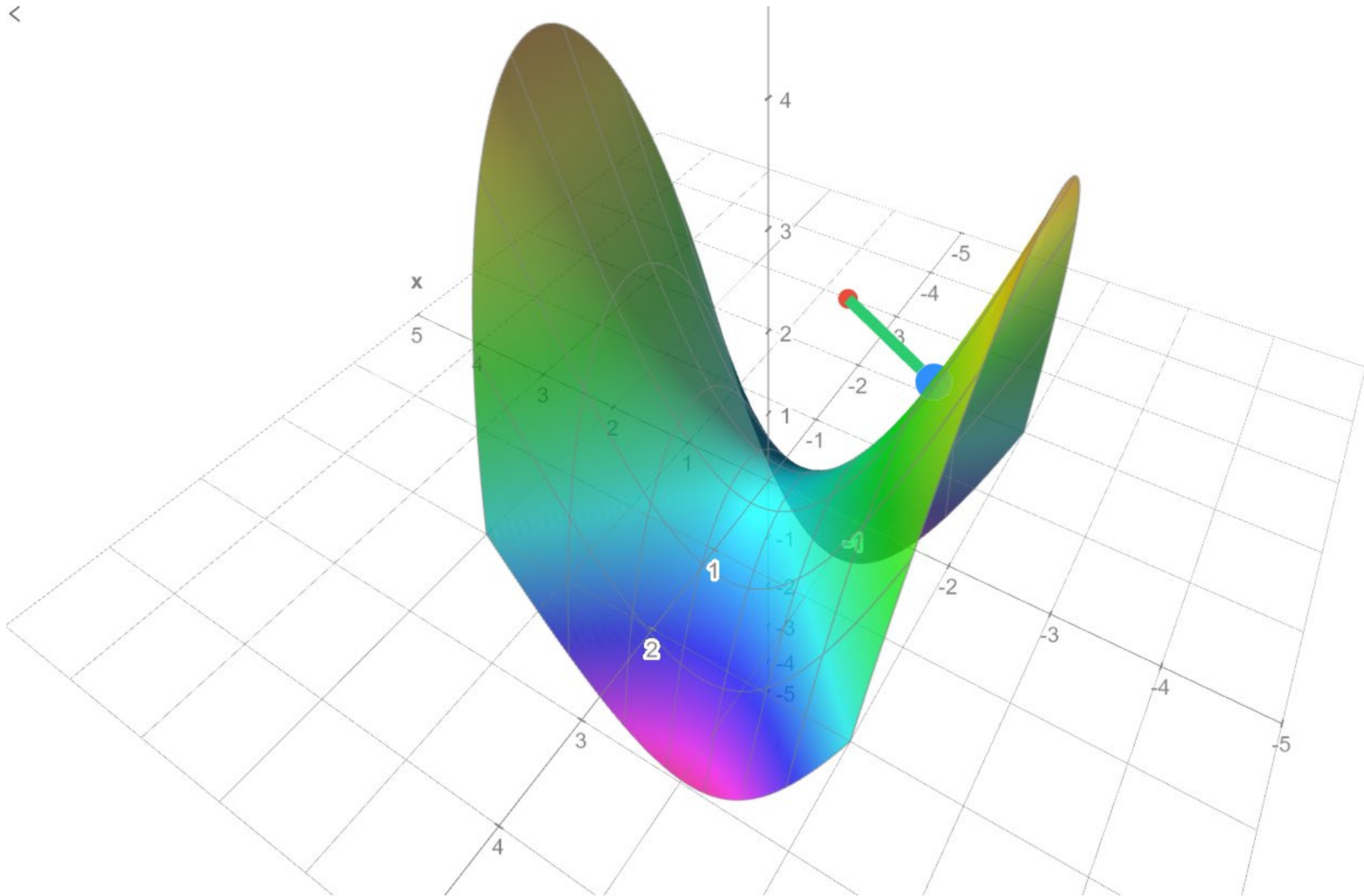


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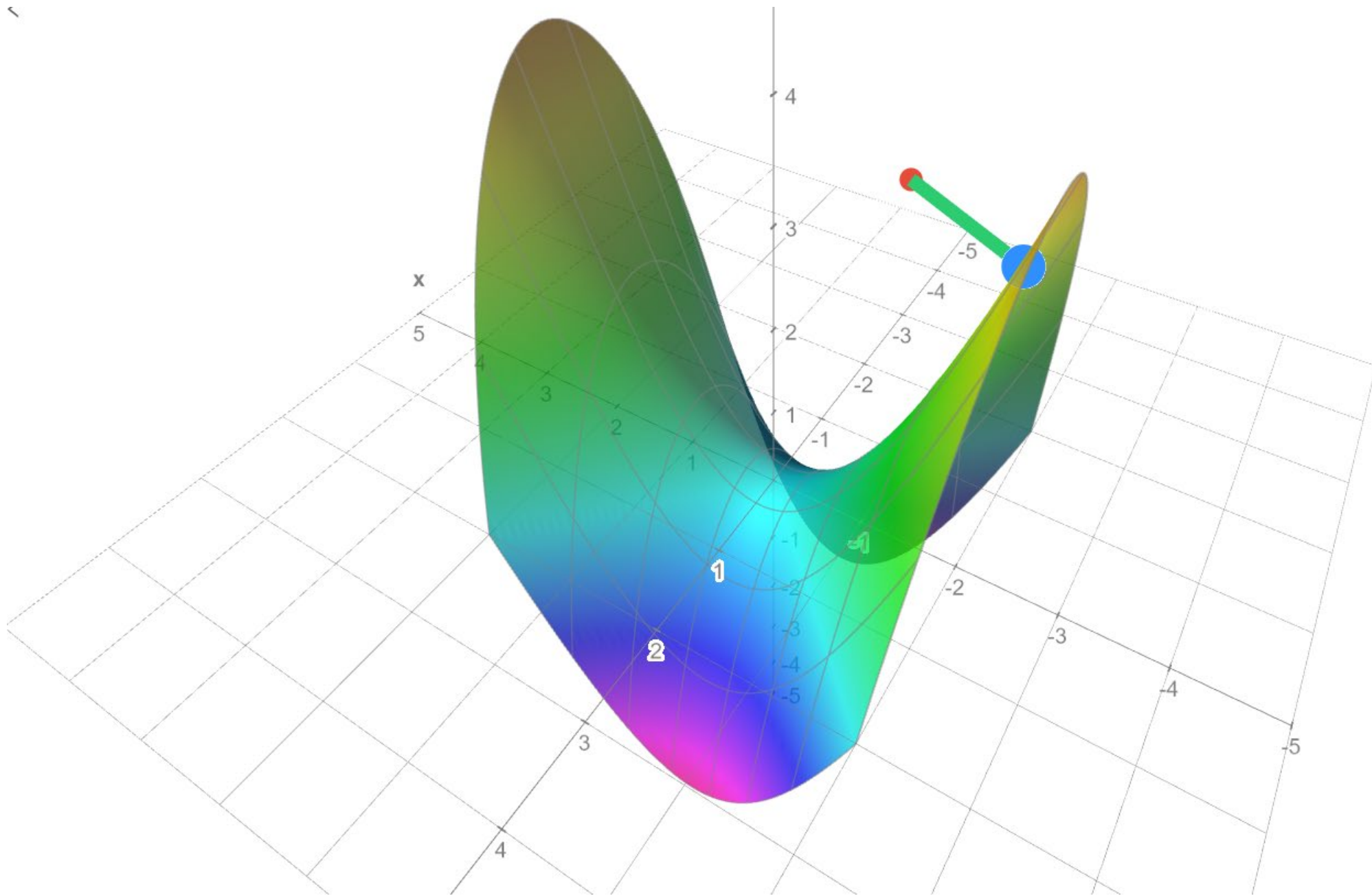




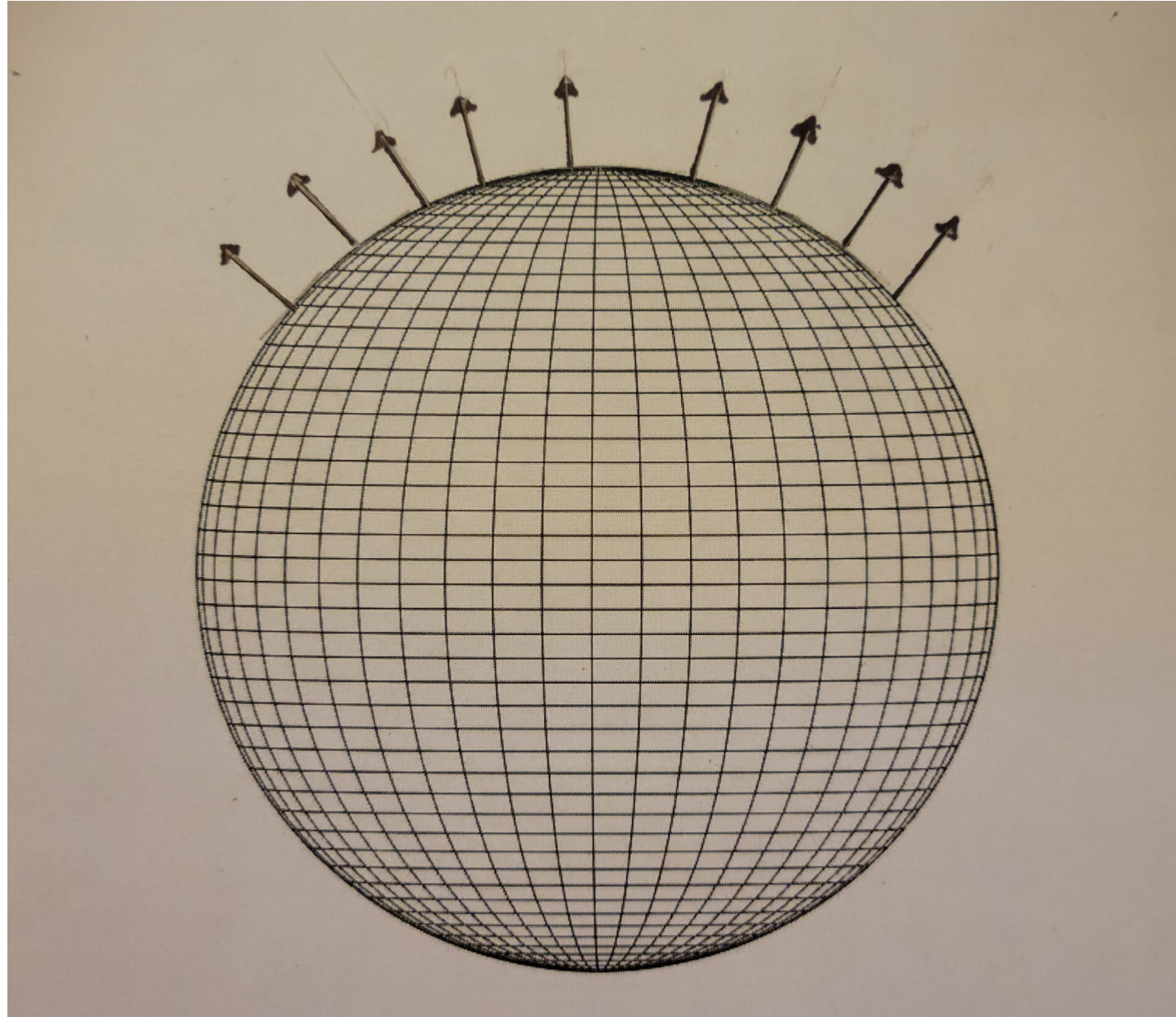
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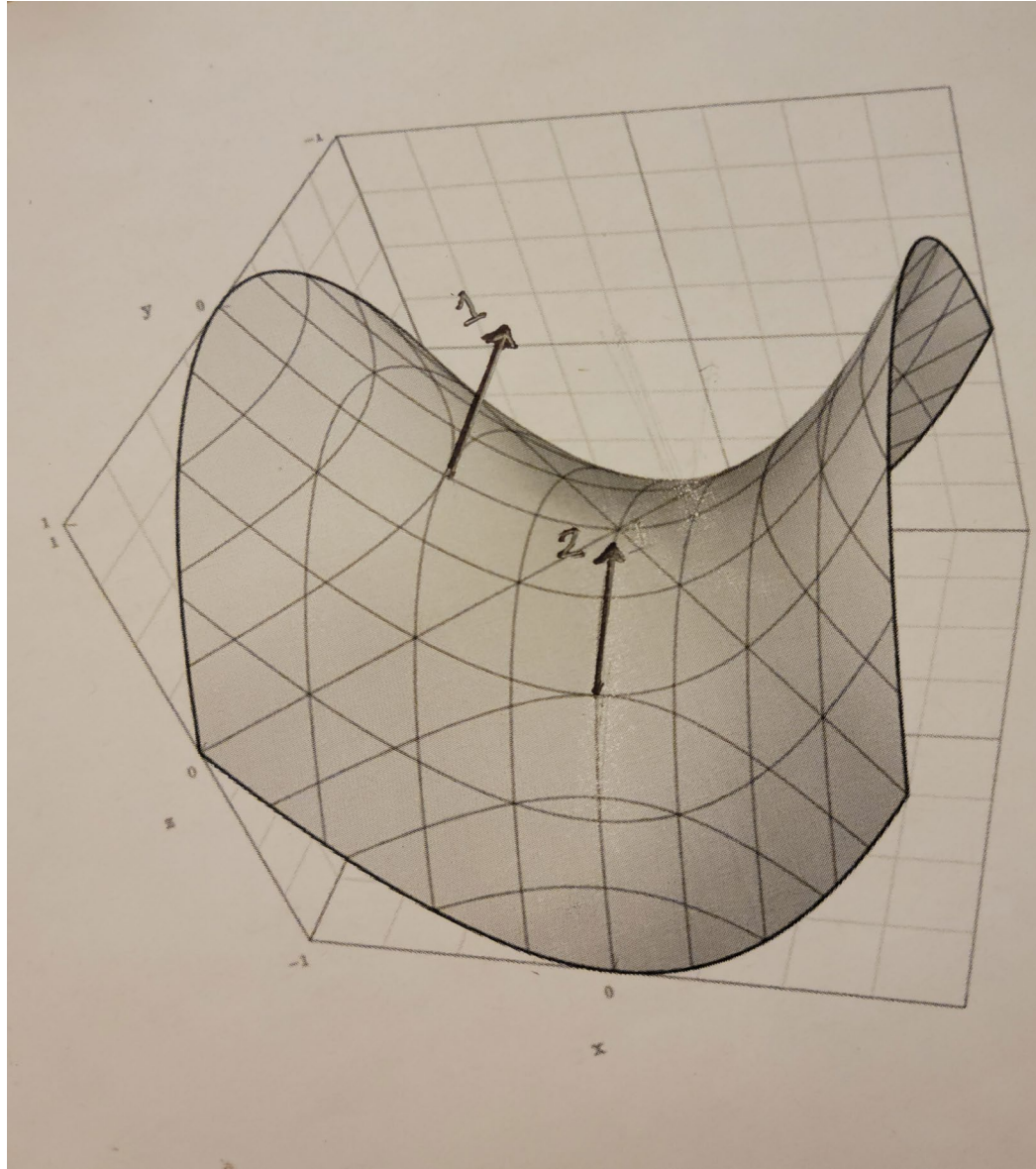
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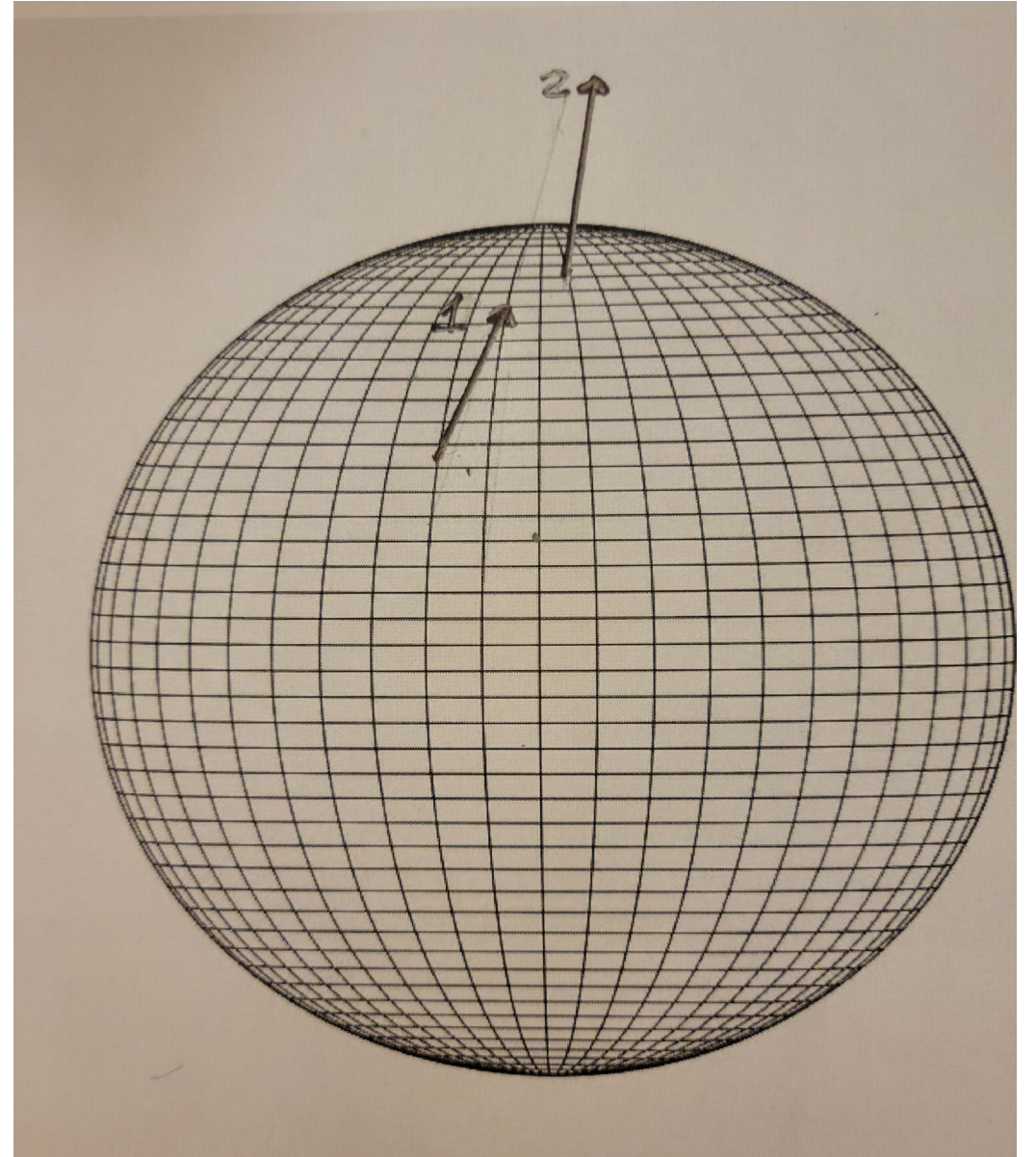
Sphere: Positive Curvature



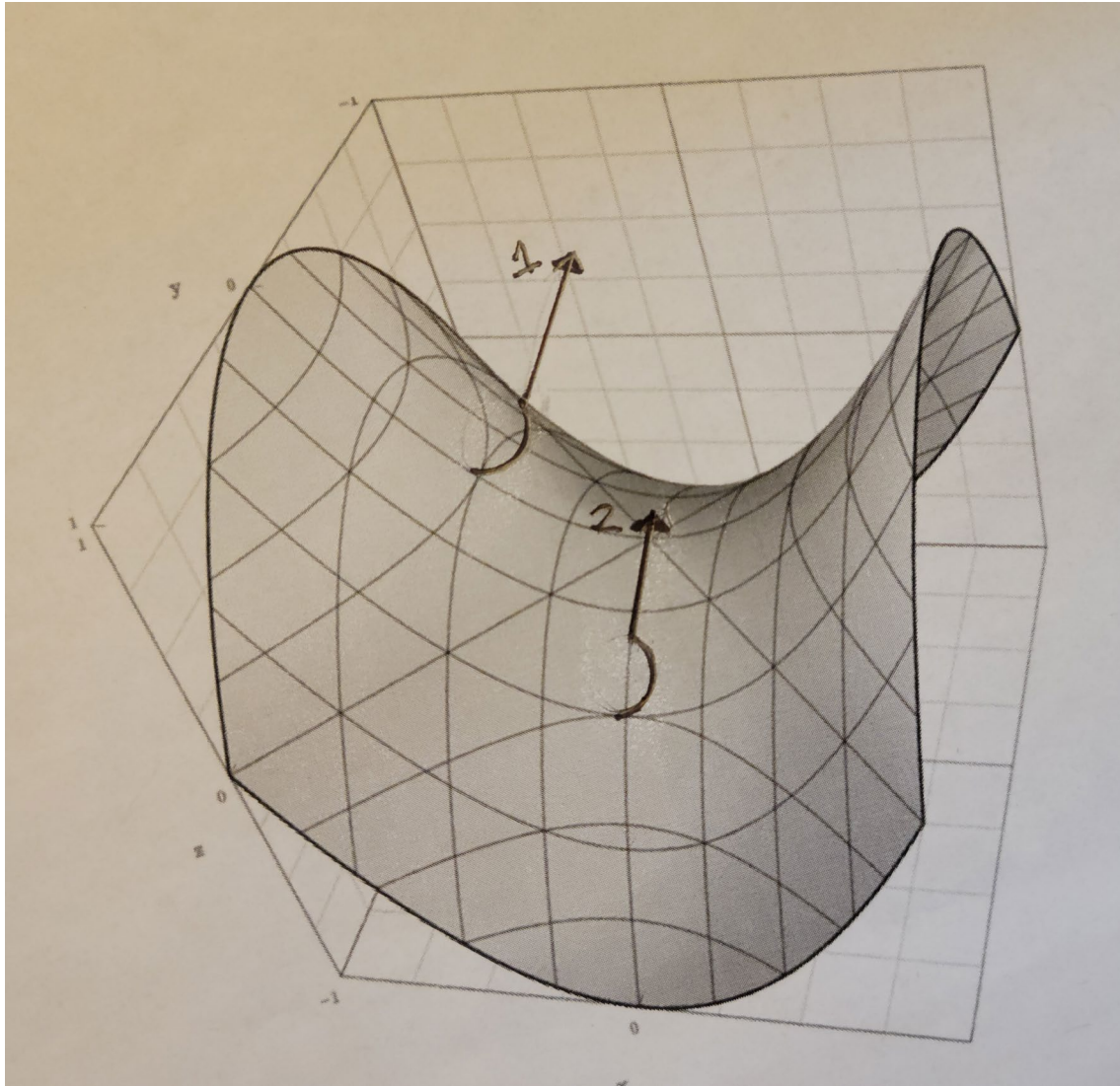
Primary Surface: Saddle



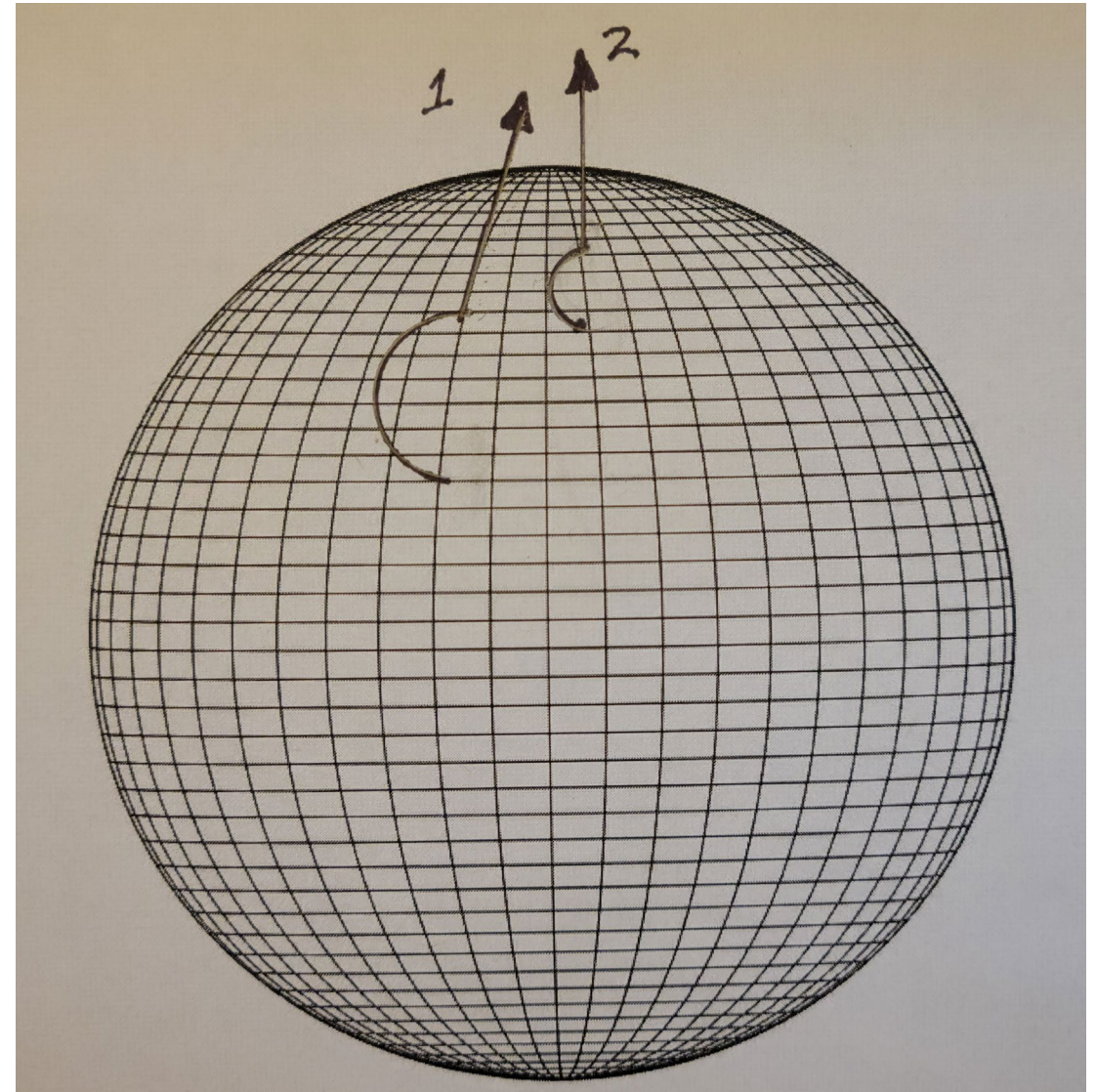
Auxiliary Sphere



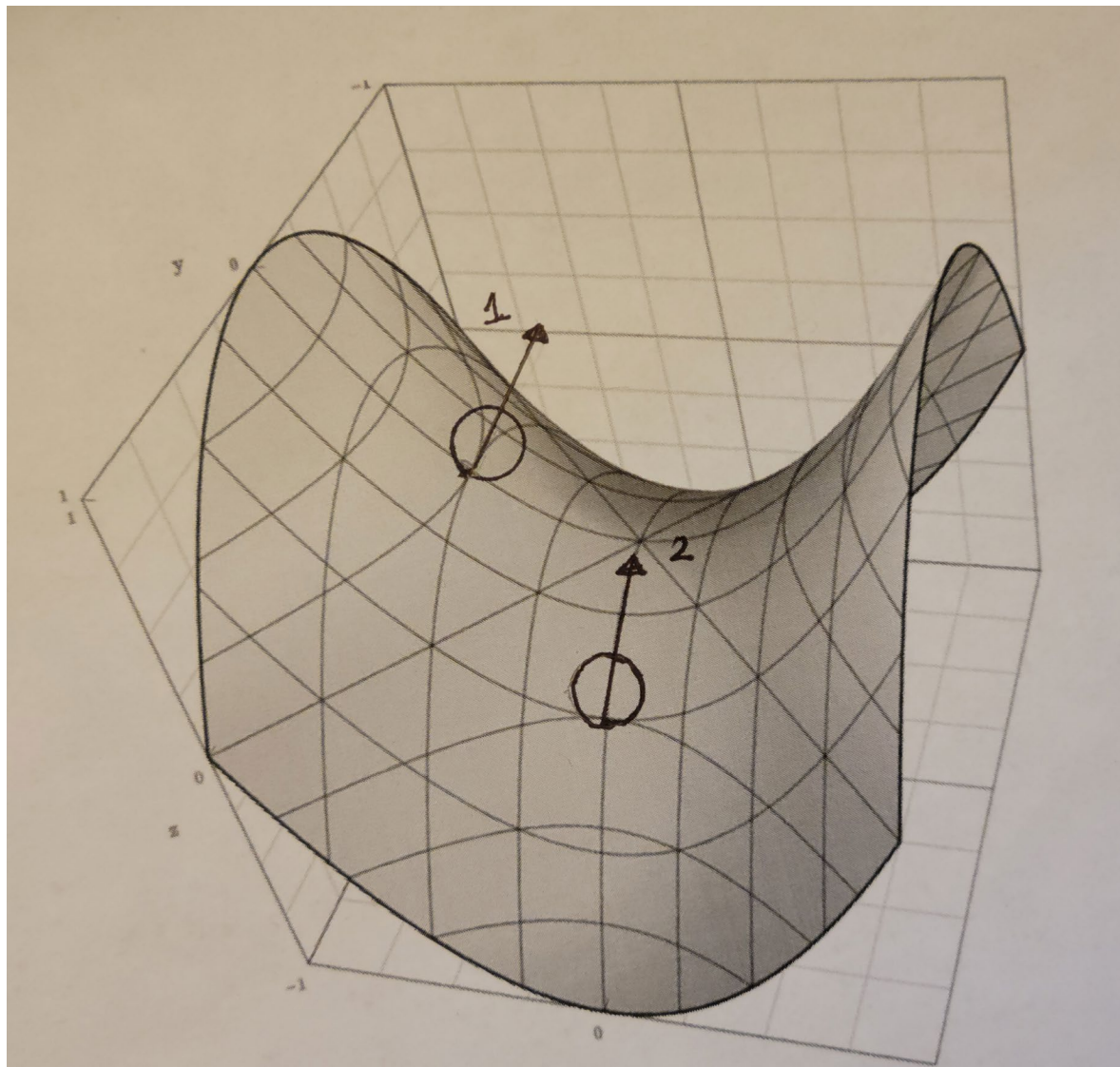
Primary Surface: Saddle



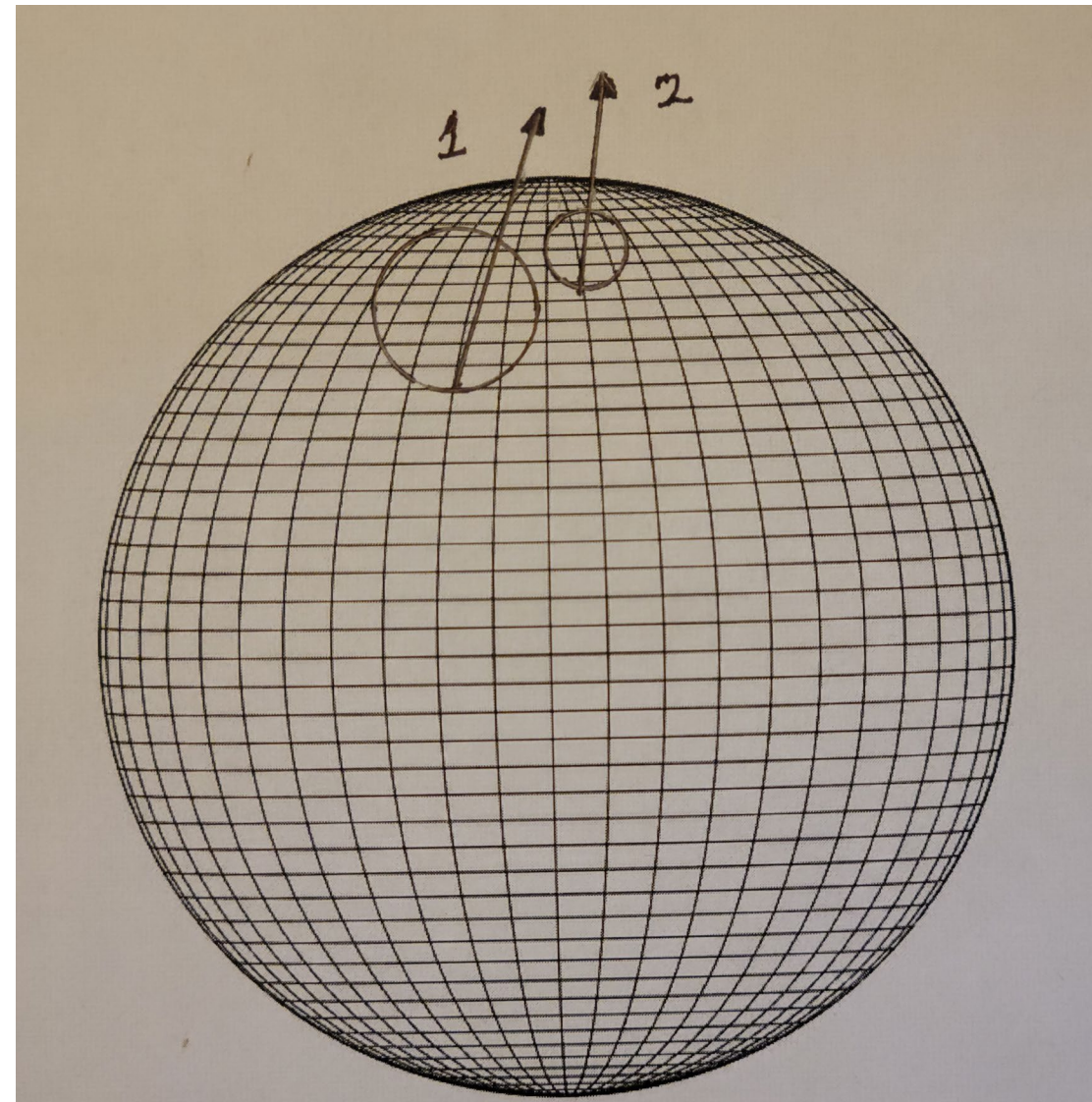
Auxiliary Sphere

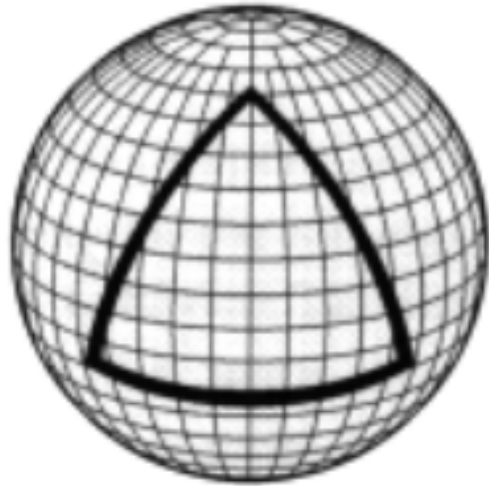


Primary Surface: Saddle

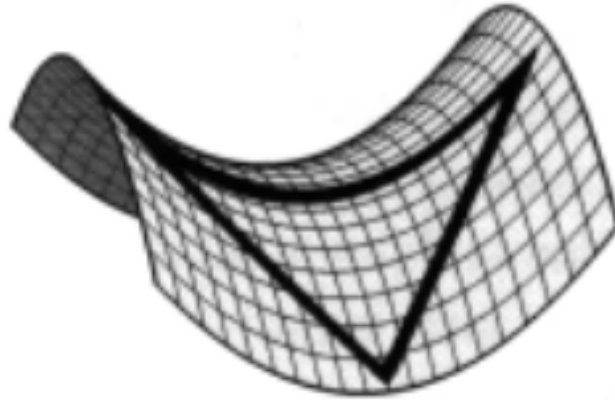


Auxiliary Sphere

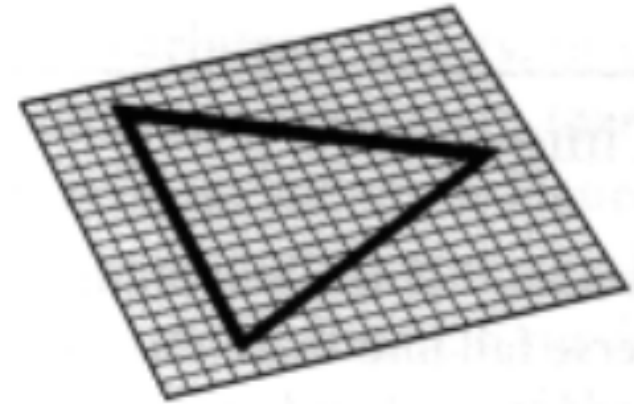




Positive Curvature



Negative Curvature



Flat Curvature

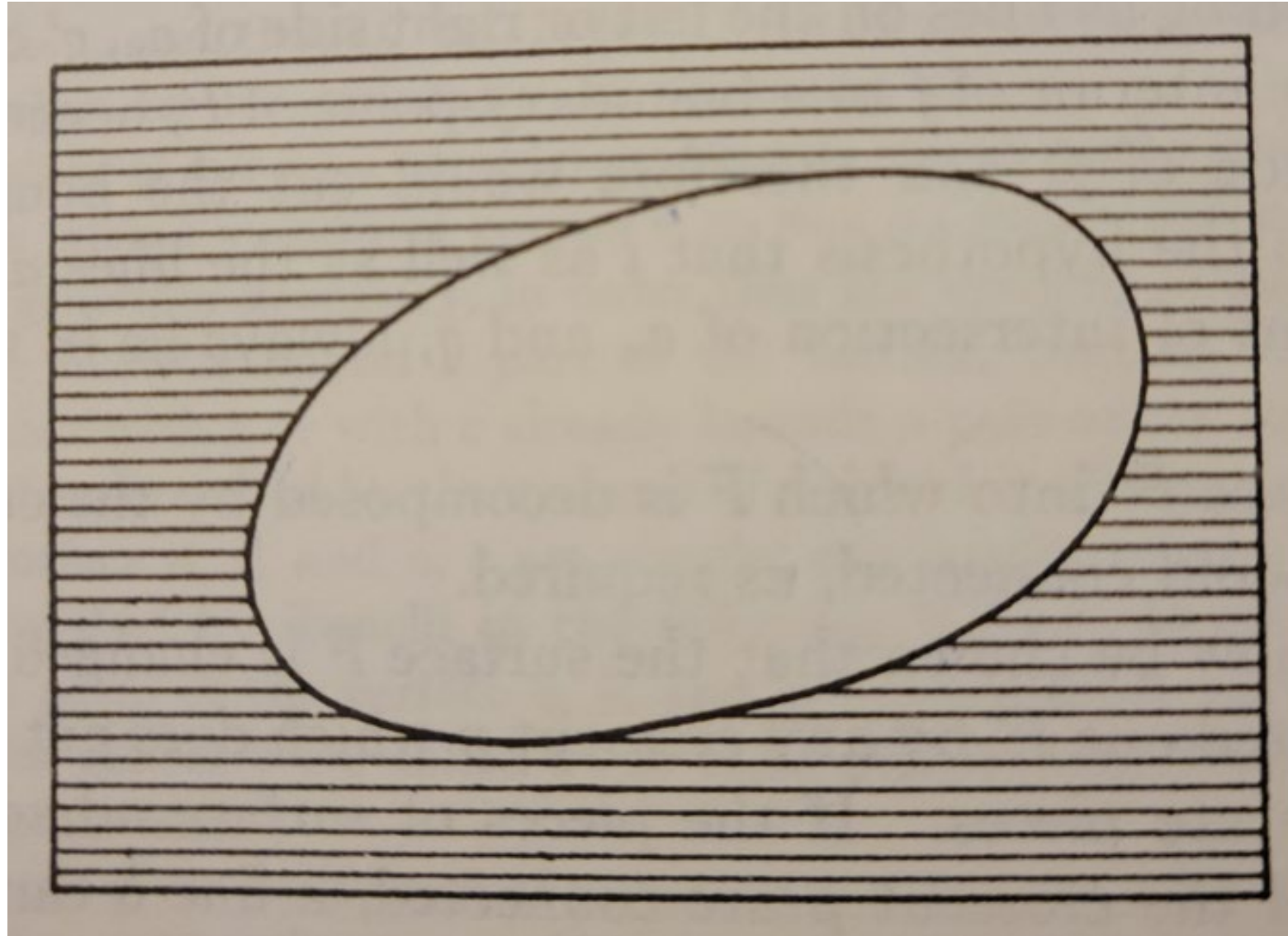
Source: Triangles on curved surfaces. Sum of interior angles of a triangle on the surface of a sphere is greater than 180° . Reprinted from: "[Geometry of the Universe](#)", J. Schombert, Univ. of Orgeon, extracted from <https://medium.com/@madeofdarkmatter/how-do-astronomers-map-objects-in-the-sky-30501d58bbb4> article by Arya, Aayush.

"Remember our review of this matter in our study of Kepler's Snowflake paper?" Positive curvature is associated with non-living functions, such as the snowflake, which do exhibit entropy as an included characteristic. However, negative curvature requires a non-entropic ordering cohering with the limiting implications of the Golden Section."

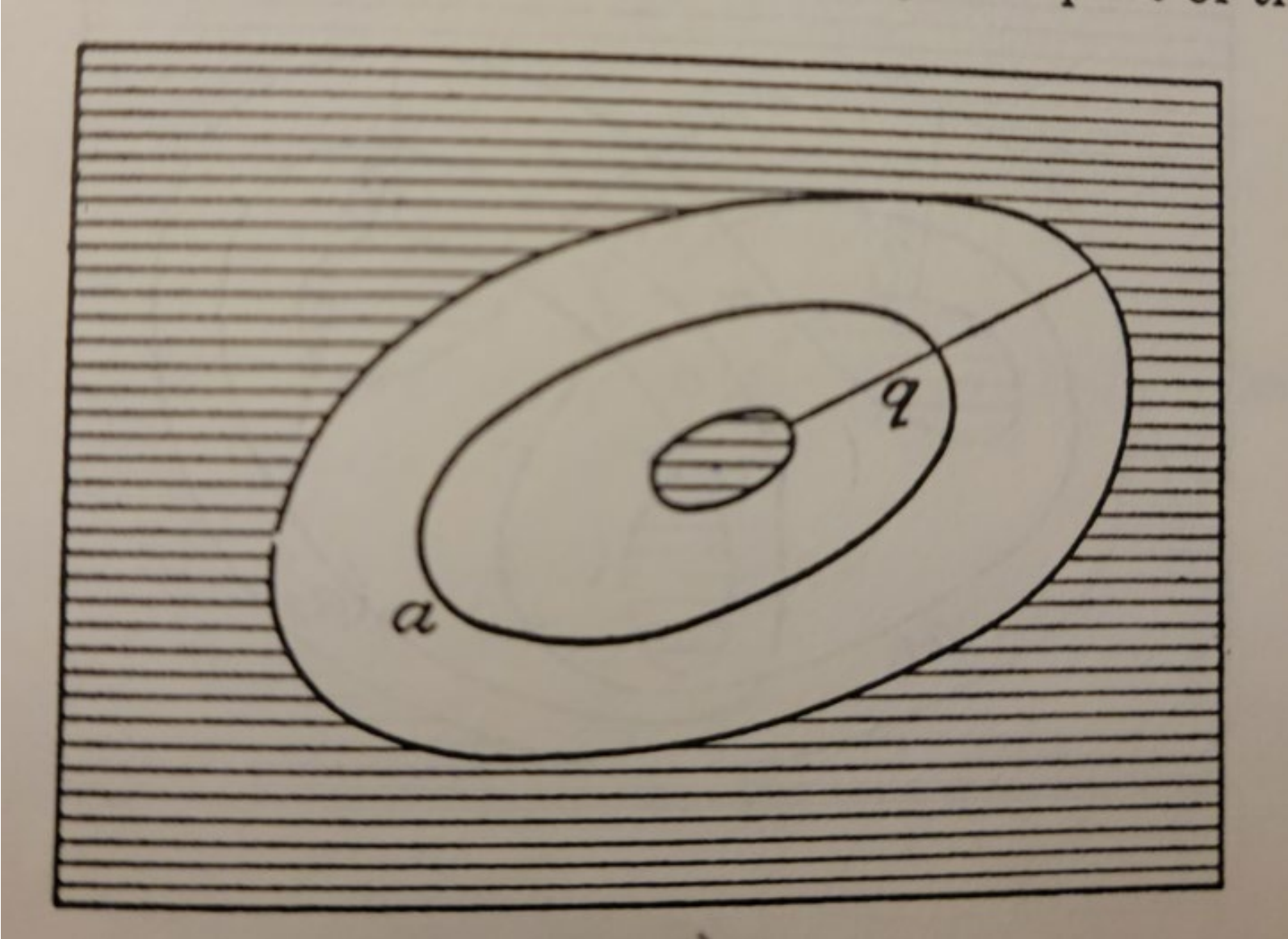
Lyndon LaRouche, "**On the Subject of Metaphor,**"

Fidelio magazine, Fall 1993

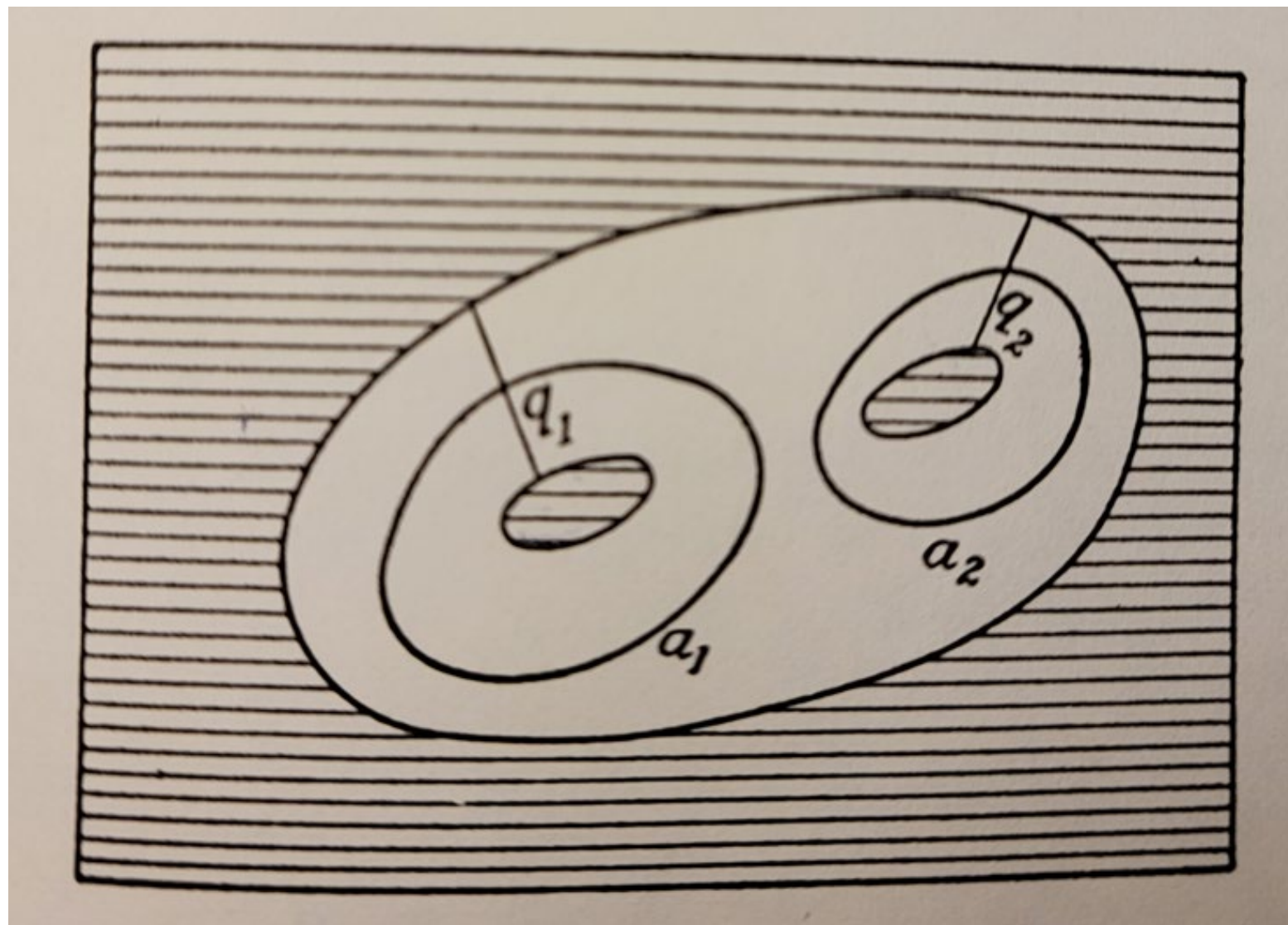
Simply Connected



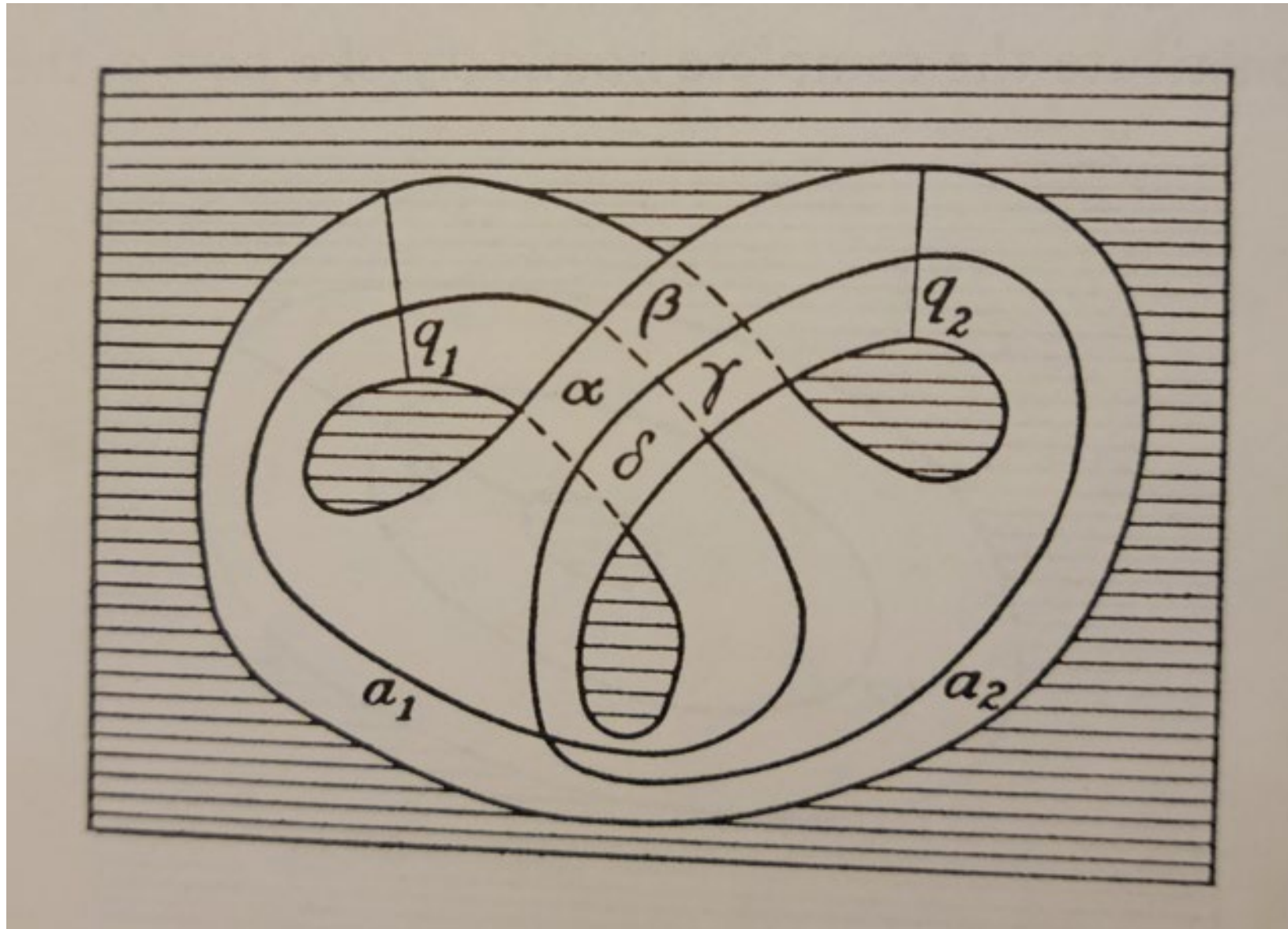
Doubly Connected



Triply Connected



Connecting the Curve

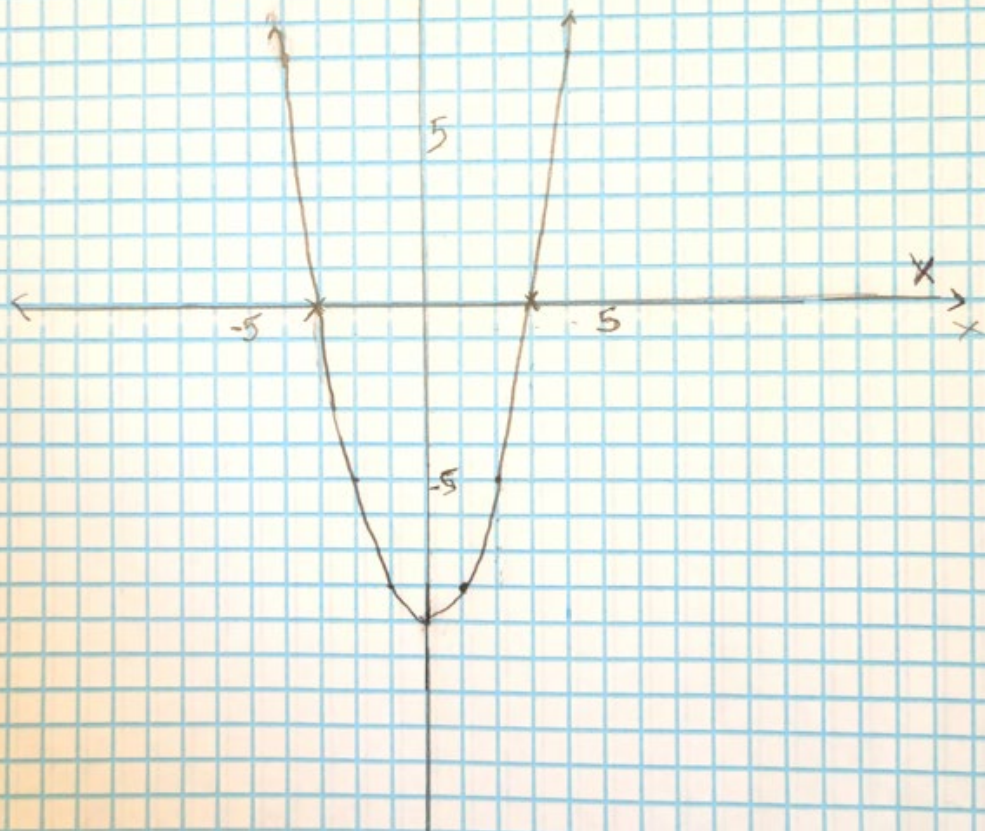


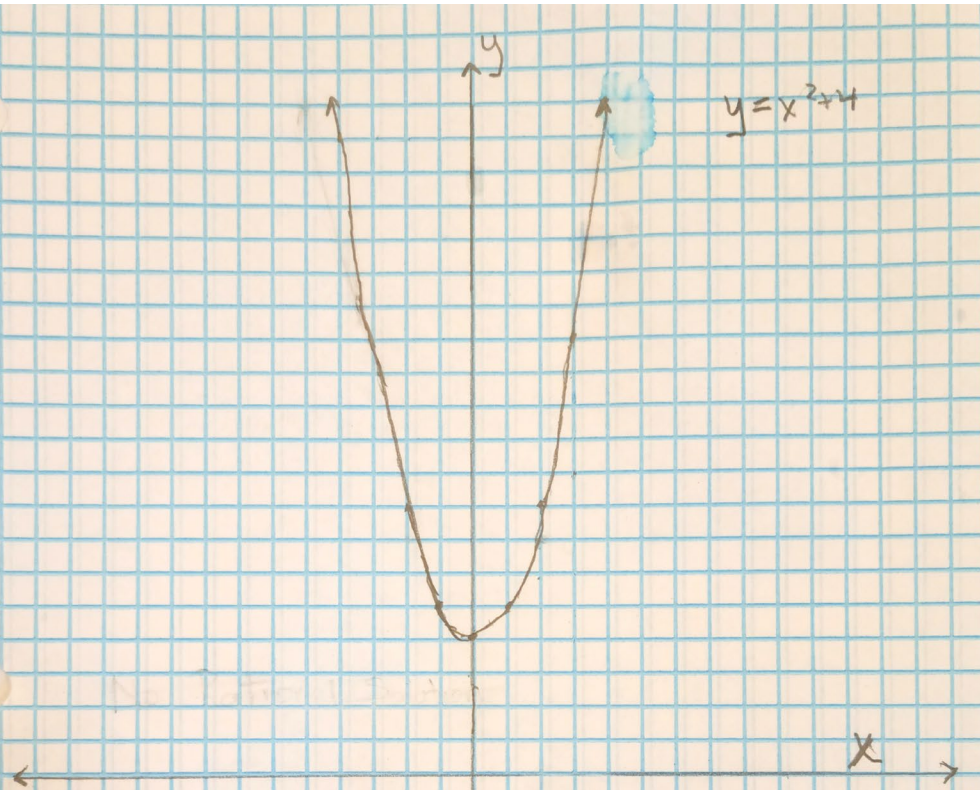
y

$$y = x^2 - 9$$

Solution of zeroes

$$x = -3, x = 3$$





$$y = x^2 + 4$$

$$0 = x^2 + 4$$

$$x^2 = -4$$

$$x = \sqrt{-4}$$

$$x = \sqrt{4} \cdot \sqrt{-1}$$

$$x = \pm 2\sqrt{-1}$$

$$x = \pm 2\sqrt{-1} = \pm 2i$$

$$x = \pm 2i$$

No Rational Solution

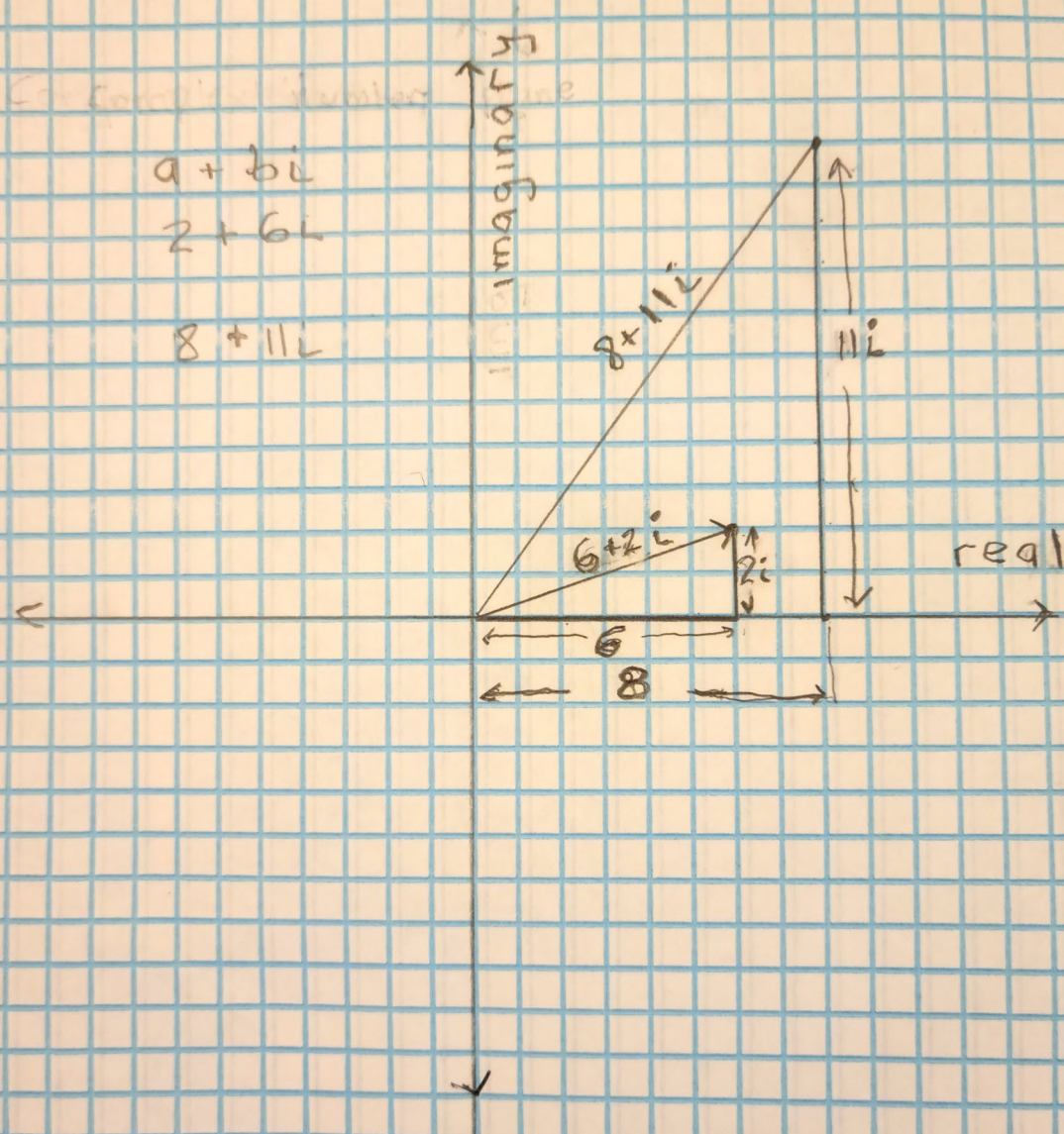
Complex Plane

Complex Number

$$a + bi$$

$$2 + 6i$$

$$8 + 11i$$



Video:

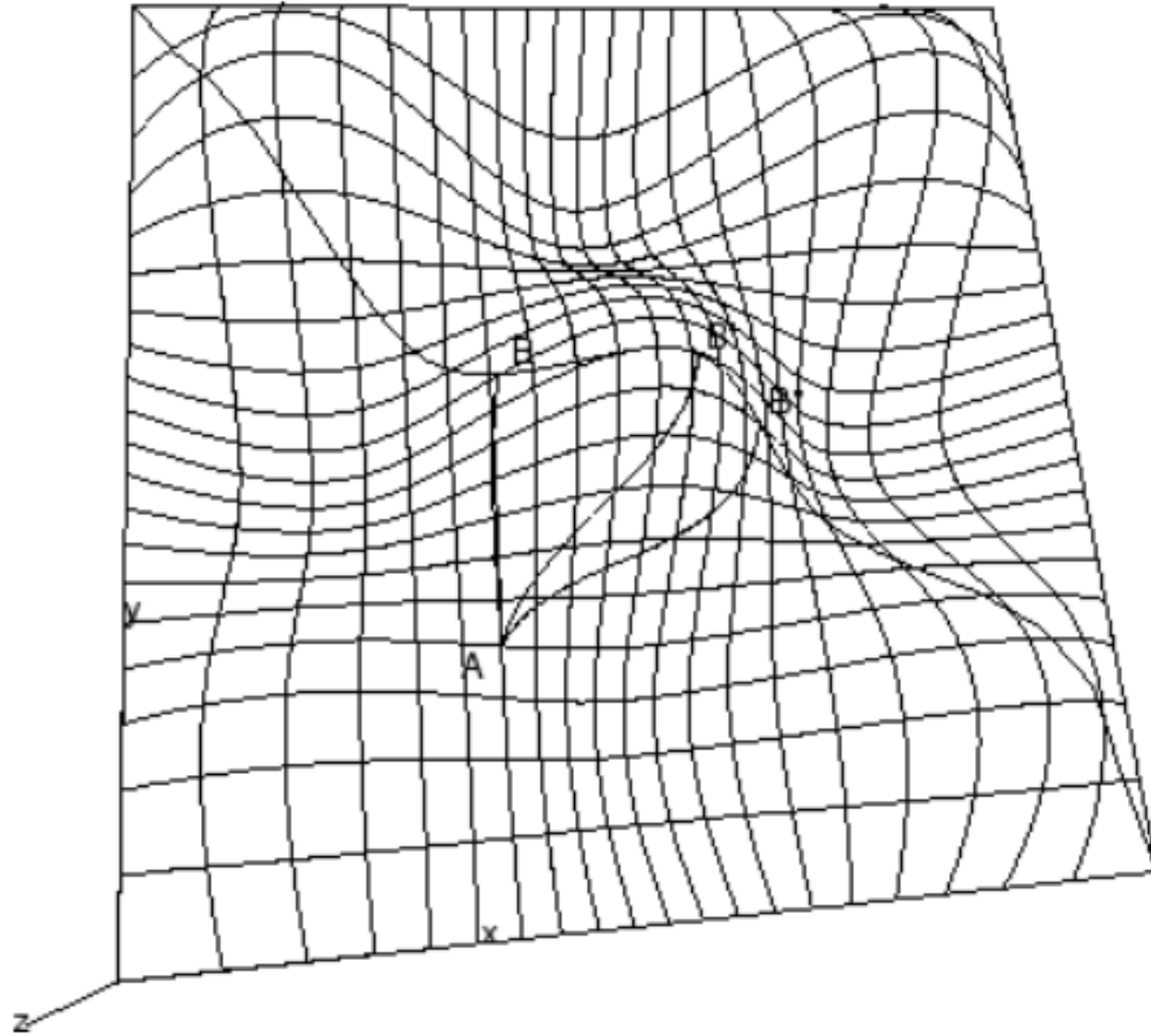
[Multiplying complex numbers as vectors](#)

Video:

[Multiplying complex numbers: \$i \times i = -1\$](#)

Video:

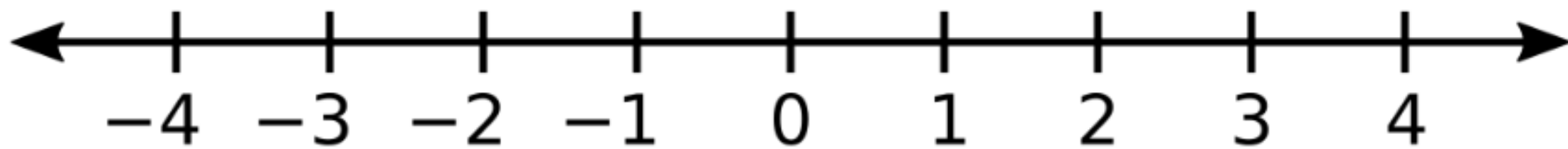
[Two layers of complex squaring](#)



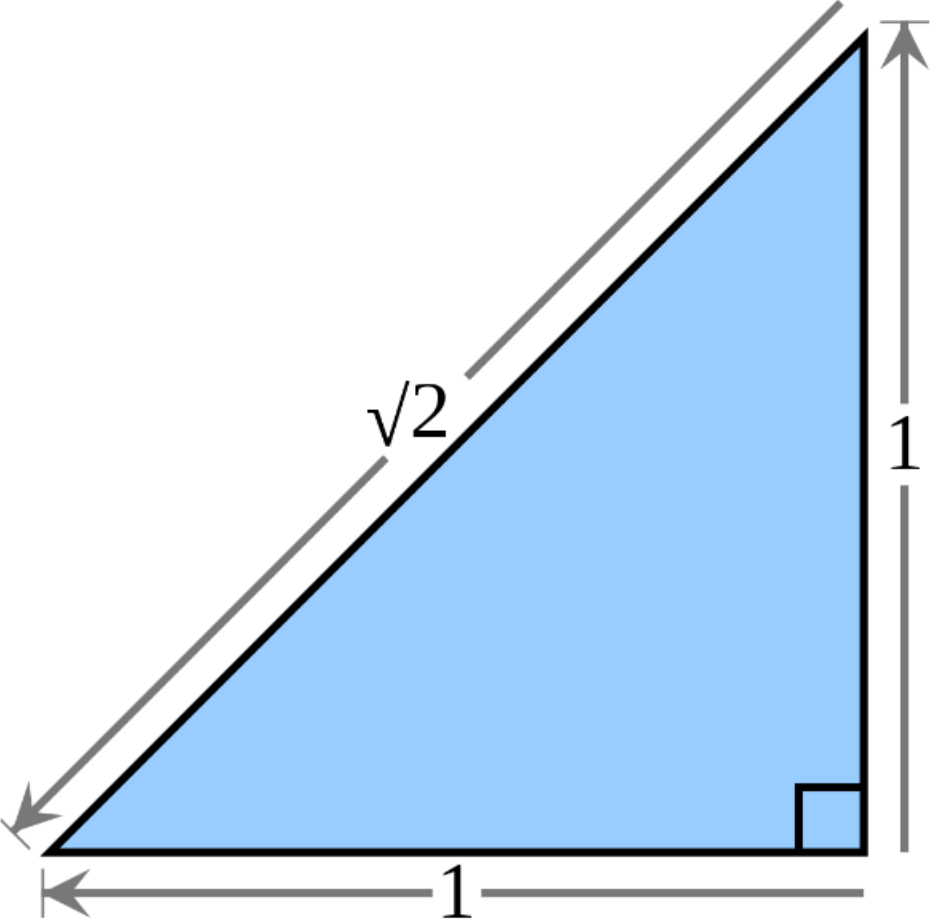
Source: 10.6.2 Geodesic Path between a Point and a Curve, web.mit.edu/hyperbook/Patrikalakis-Maekawa-Cho/node207.html. Accessed 3 Aug. 2023. Adapted from T. Maekawa Computation of Shortest Paths on free-form parametric surfaces, *Journal of Mechanical Design Transactions of the ASME*, 118(4):499-508, December 1996

Geodesic is Shortest Distance On Earth

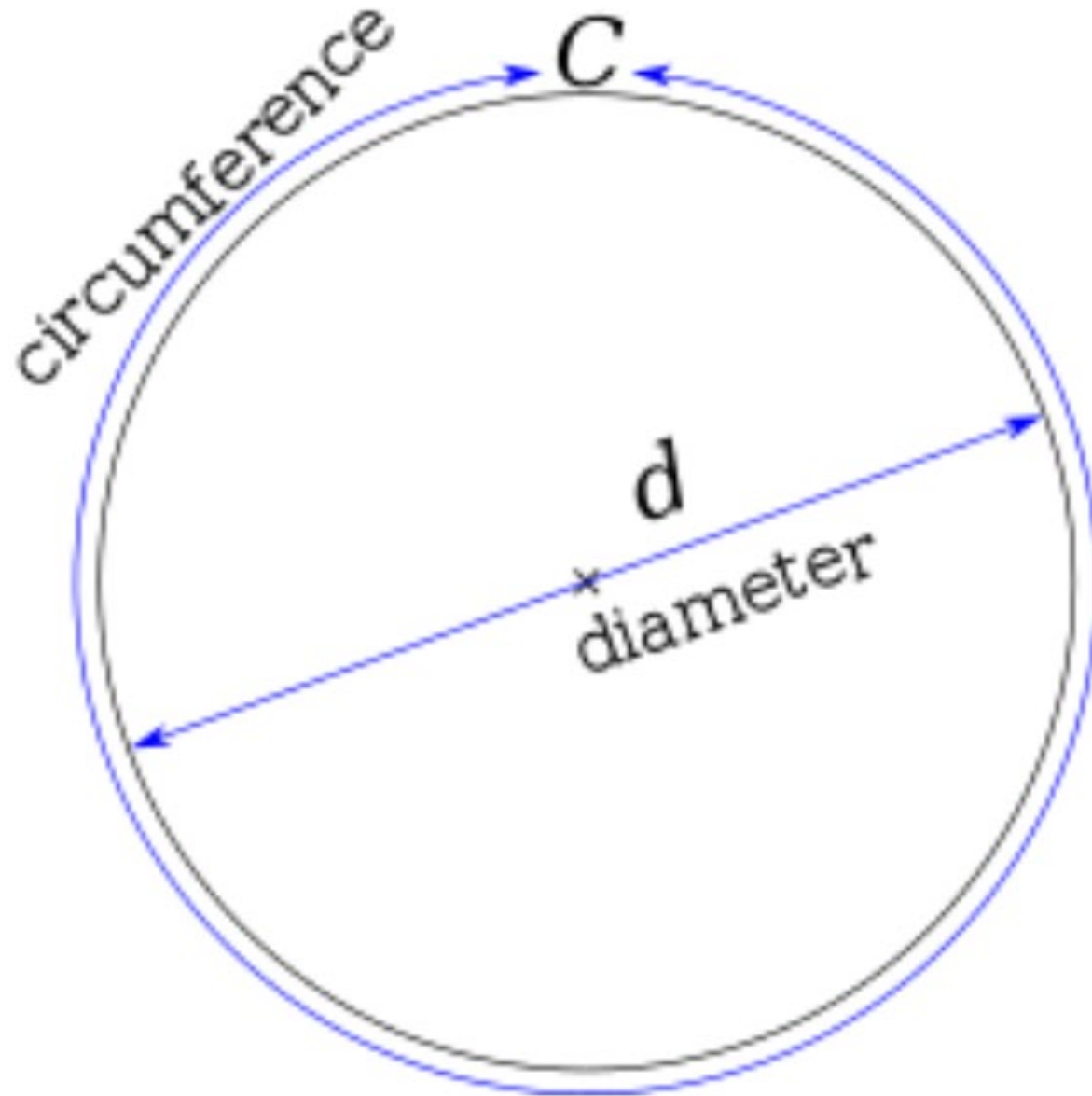


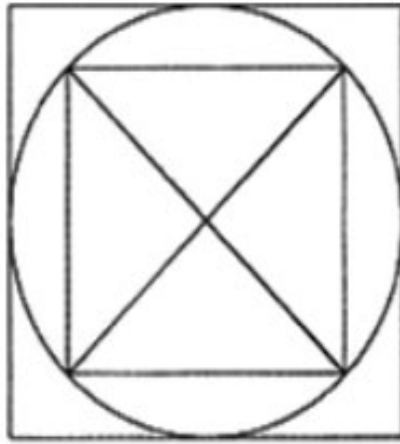


Square Root of 2 Is Constructed In Space of Two Dimensions

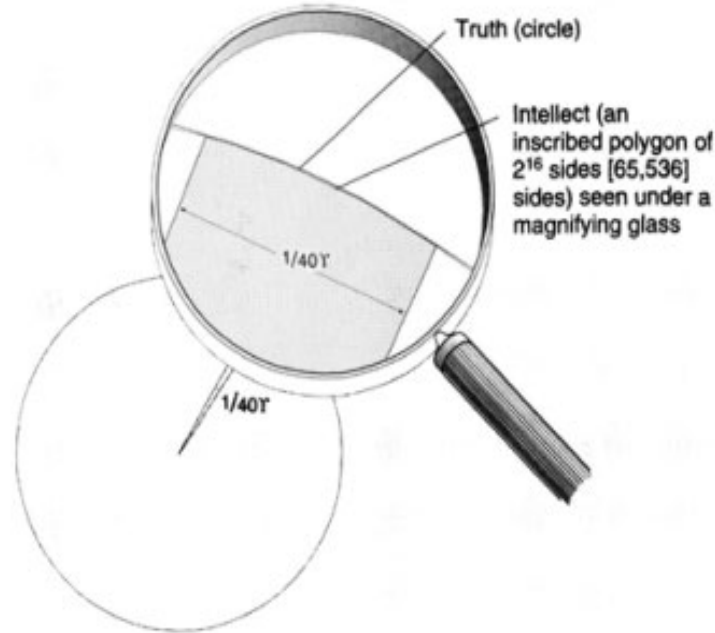
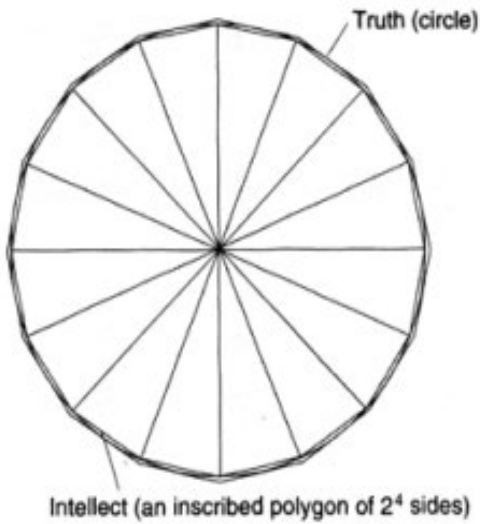


π Relation of Circumference to Diameter, C/D

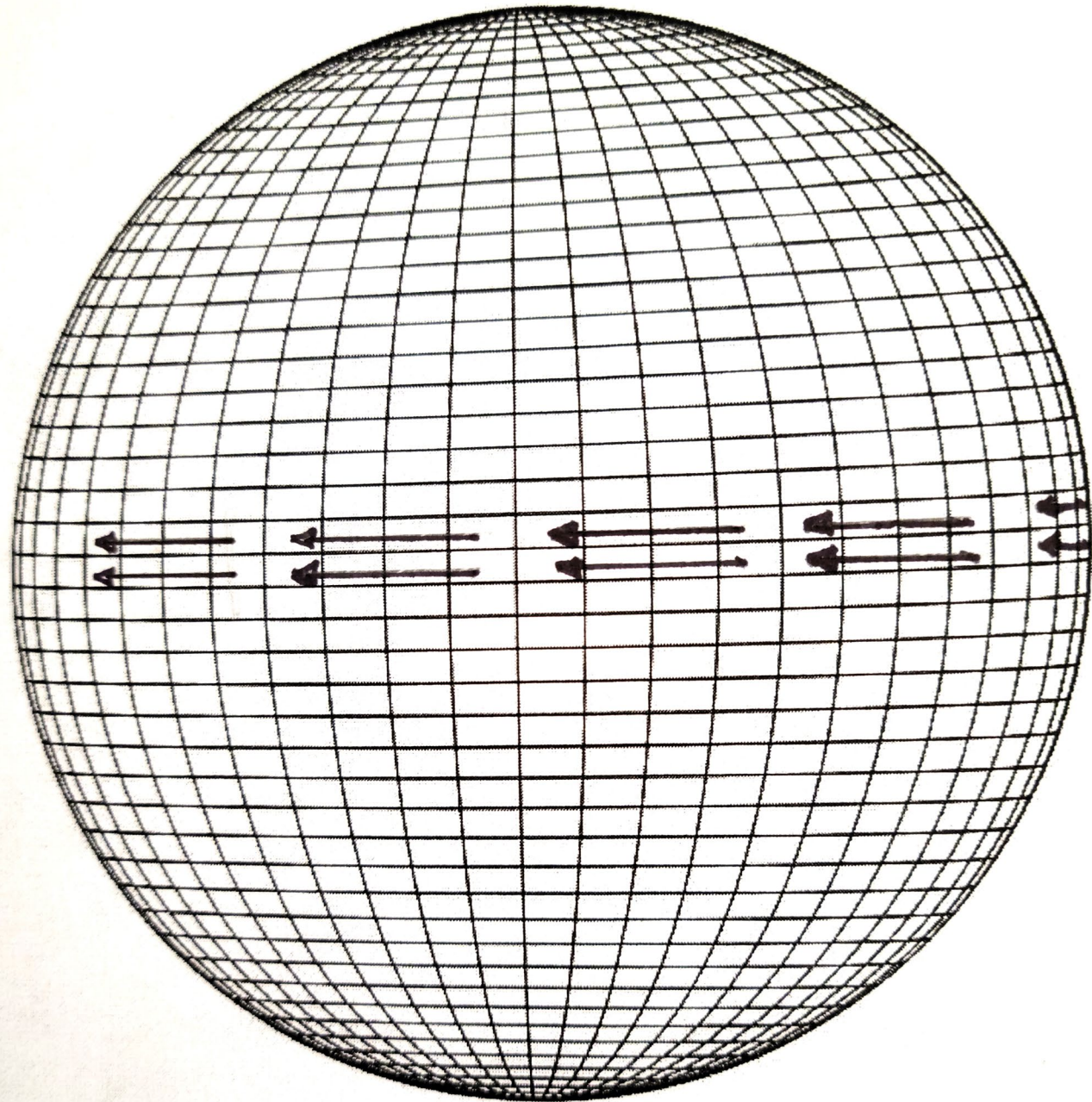




“The intellect is to truth, as an inscribed polygon is to the inscribing circle. The more angles the inscribed polygon has, the more similar it is to the circle. However, even if the number of angles is increased ad infinitum, the polygon never becomes equal to the circle.”



Nicholas of Cusa (1401-1464)



The universe is multiply-connected in the Riemannian sense. This has the essential implication, that the addition of a new principle to the manifold results in a change in the characteristic “curvature” of the manifold as a whole. This means, that the value, the impact of each such discovery, changes the value of each previously established type of action within the thus-unfolding manifold. So, for example, a successful such change within the technology of a society, will tend to increase the effective productivity of even those forms of action which are not themselves changed otherwise. So, a genuine improvement in the basic economic infrastructure of a society, increases the productivity of otherwise unaltered modes of productive action within the society as a whole. By definition, no mathematical model of the Descartes-Newton type could represent such connections and effects.

Lyndon H. LaRouche, **“A Lawless USA Today: Faith, Hope, And Agapé!”**

May 13, 2001

Succession of LaRouche-Riemann Manifolds

$n + 2$



$n+1$



n

The central feature of my original contribution to the Leibniz science of physical economy, is the provision of a method for addressing the causal relationship between, on the one side, individuals' contributions to axiomatically revolutionary advances in scientific and analogous forms of knowledge, and, on the other side, consequent increases in the potential population-density of corresponding societies. In its application to political economy, my method focuses analysis upon the central role of the following, three-step sequence: first, axiomatically revolutionary forms of scientific and analogous discovery; second, consequent advances in machine-tool and analogous principles; finally, consequent advances in the productive powers of labor.

These discoveries were initially the outgrowth of 1948-1952 objections to the inappropriateness of Norbert Wiener's application of statistical information theory to describing both the characteristic distinctions of living processes and of communication of ideas.¹ I countered with a contrary, non-statistical definition of negentropy, as that meaning of the term might be derived from the common, physically distinguishing characteristic of an evolutionary biosphere.

> > >

This nonstatistical counter-definition of negentropy was then stated in terms of a successfully self-developing physical economy; the efficient impact of scientific discoveries' communication within such a negentropic physical-economic process was treated as most typical of the communication of ideas in general.

That was the initial core of my discovery, up to the year 1952. Yet, up to that point, the appropriate mathematical representation of such a form of physical-economic negentropy was still wanted. . . .

On LaRouche's Discovery

by Lyndon H. LaRouche, Jr - November 21, 1993

My 1952 study of Cantor's *Beiträge* provided the key to developing this conception further. Following that study, later the same year, I was electrified by rereading the relevant, most crucial passage of Riemann's habilitation dissertation.²⁴ Applying the Cantorian implications of my own notion of negentropy to Riemann's stated crucial problem of a continuous manifold "sent sparks flying in all directions." Cantor's transfinite was key to bringing the two elements together in this way, my own and Riemann's.

On LaRouche's Discovery

by Lyndon H. LaRouche, Jr - November 21, 1993

Initially, during 1948-1952, I made two principal arguments against Norbert Wiener's application of statistical method to living processes. The first of these two was, that, insofar as we employ the term "negative entropy" to signify the characteristic distinction of living processes in general, the phenomenon referenced cannot be described either in terms of a simple time reversal of thermodynamical statistical entropy, or in terms of the term "energy" used as a notion reducible to a scalar measure of heat. . . .

On the first of these two classes of objections, the kernel of the matter is, that, for the case of an indefinitely successfully self-developing biosphere, the imputable ratio of free energy to energy of the system increases at the same time that the total energy of the system increases, and, that, similarly and concurrently, the ratio of free energy to rising energy-flux density is also rising.

On LaRouche's Discovery

by Lyndon H. LaRouche, Jr - November 21, 1993

Only the evolutionary model of such a heat- powered process of increase of the productive powers of labor brings the meaningful issues into focus. By contrast, any zero-growth, non-evolutionary model of physical economy is axiomatically entropic, and corresponds to no durably successful model of national or global economy.

For the evolutionary case, progress in scientific and analogous forms of knowledge is the driver of those changes in practice which lead toward a consequent expression of the indicated, life-like negentropic forms of economic development. It should be stressed, that this role of generation and communication of ideas is illustrated by considering Leibniz's study of the proposals for an industrial development based upon the combination of heat-powered machinery and analogous thermodynamical development of modes of production and transport generally. This Leibniz case is a bench-mark from which the history of physical economy in general may be traced backward and forward in time.

> > >

That Leibniz case, of increase of the productive powers of labor through employment of the heat-powered machine, has two readily identified, ironically juxtaposed aspects. First, immediately, there is the simpler aspect, the increase of productive powers of labor, in some functional correlation with increase of heat power supplied efficiently per capita and per square kilometer. In the complementary aspect, on account of nothing other than some improvement in employed principles of design, one machine, using no more power than a comparable second machine, yields greater increase of the productive powers of labor. The second case, the general notion of an efficient improvement in design principle, illustrates the notion of technology.

On LaRouche's Discovery

by Lyndon H. LaRouche, Jr - November 21, 1993

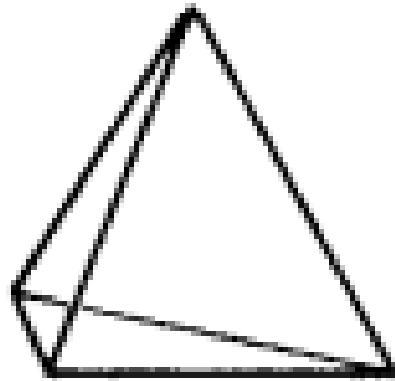
The characteristic, absolute superiority of our human race over all lower species, is expressed implicitly by mankind's rise from a bestial, baboon-like, rockart-like potential population density of circa ten millions living individuals, to a technologically-determined potential of more than twenty-five billions today. This change is owed entirely to a quality which the Christian's Latin terms *imago Dei* and *capax Dei*, the Mosaic tradition of Genesis 1, that man, male and female alike, is cast in the image of God. This likeness is by virtue of that power of creative reason which is most simply illustrated by a revolutionary-axiomatic superseding of inferior by superior principle of scientific practice.³⁹ Thus, in effect, mankind is the only super-species, the only species which can willfully self-develop itself to the physical-economic equivalent of a succession of successively higher species.

To state this pivotal point very briefly, this quality of being such a "super-species" of creative reason is the image of negentropy as far as the human mind is capable of defining that notion. As such a "super-species," insofar as our physical-economic practice is premised upon such a continuing process of science-driven increase of our power of physical-economic practice, per capita and per square kilometer of our earth's habitable surface,⁴⁰ our conscious reflection upon our revolutionary practice is this idea of negentropy, this notion of the ontologically transfinite.

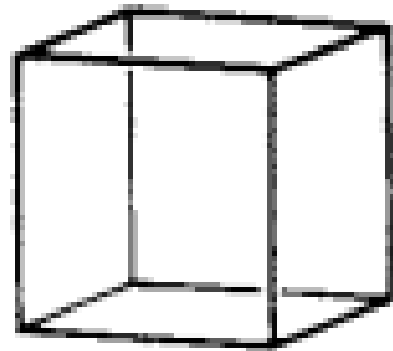
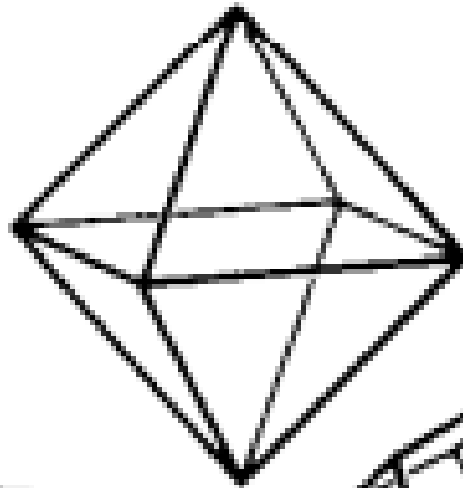
On LaRouche's Discovery

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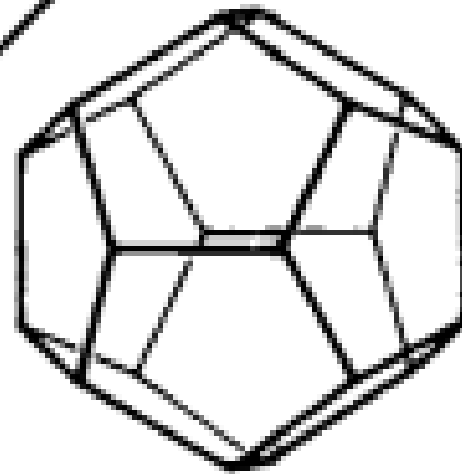
Tetrahedron



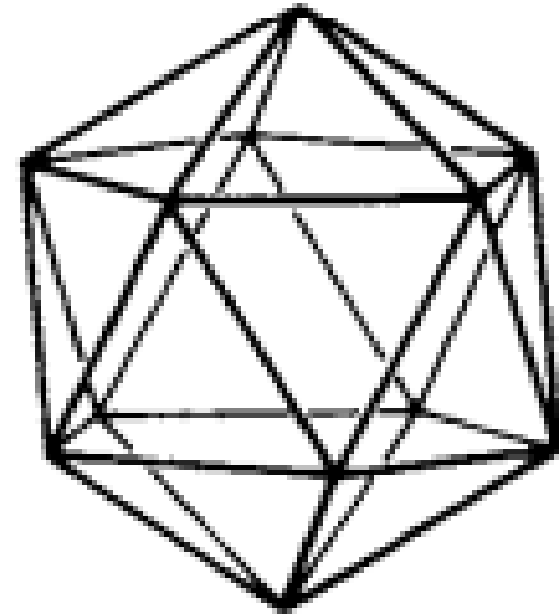
Octahedron



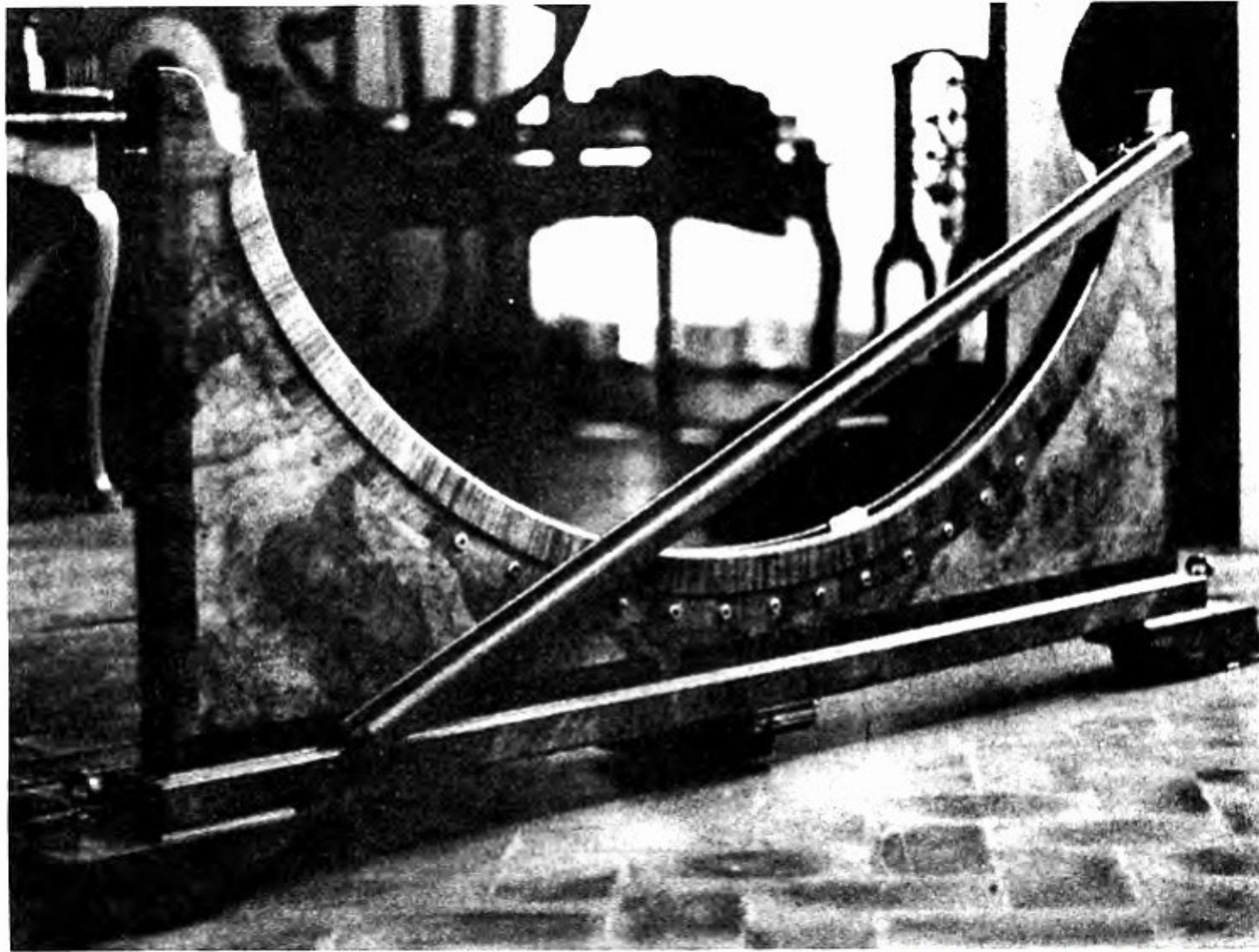
Cube



Dodecahedron



Icosahedron



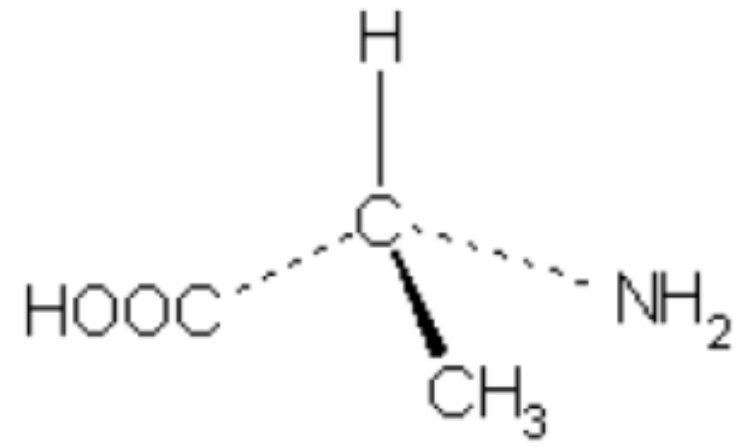
Museo di Storia della Scienza, Florence

An 18th century brachistochrone.

Catenary: Hanging Chain

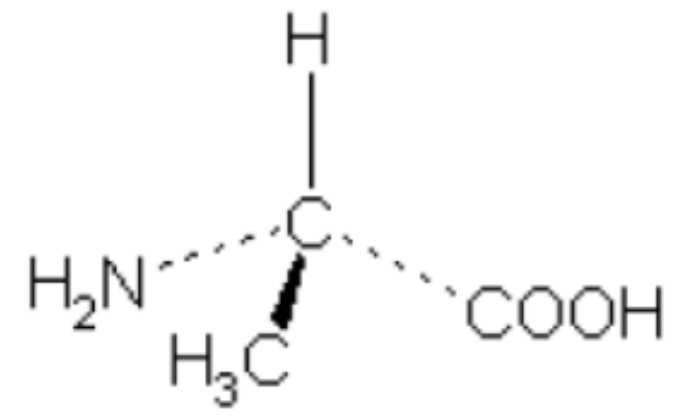


Chirality—Handedness



D-Amino acid

Left-Handed



L-amino acid

Right-Handed

Any master's Classical composition according to the principles of motivic thorough-composition, such as those of Wolfgang Mozart, L. v. Beethoven, F. Schubert, R. Schumann, Johannes Brahms, et al., must be performed by applying the developed conception reached at the close of the composition, to the interpretation of every portion of the composition, from the beginning of the performance of the composition.

**The Essential Role of 'Time Reversal' In Mathematical Economics
by Lyndon LaRouche, Oct. 11, 1996**

In mathematics and physics, a “Platonic idea”; appears only as cognitive mental activity within the mind of either an original discoverer, or, of a student who comes to know that idea in the only way possible, through replicating the mental act of original discovery within the confines of the student’s own, sovereign mental processes. In both cases, original discoverer, or student, knowledge can not be obtained by mere classroom and textbook learning of the means to pass an examination, such as that idiot-savant’s delight, the multiple-choice questionnaire; it must be acquired by the kind of deductively-discontinuous mental processes unique to generating an original discovery. In the lesser case, the Platonic idea appears as the initial act of discovery of a theorem which is consistent with an implicitly preexisting hypothesis. In the higher-ranking case, the same method of original discovery is the means by which the discovery of new axioms (e.g. , a superior hypothesis) is accomplished.

As Riemann introduces this notion in his 1854 habilitation dissertation: The interdependent issues of hypothesis and of physical space-time curvature become unignorable in mathematical physics, whenever an experimental paradox compels us to introduce a validated new principle of experimental physics.

**The Essential Role of ‘Time Reversal’ In Mathematical Economics
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However, every hypothesis, or higher hypothesizing, acts simultaneously upon every possible element of sequence within the domain which that hypothesis underlies.”

In Riemann’s Platonic, Leibnizian physics, every discovered principle of nature which is validated by the methods of experimental-physical measurement specified by Cusa, functions, like spatial extension and time, as an extensible dimension of a general physical-space-time manifold. With each validated addition of such a dimensionality, we are obliged to validate, by experimental measurement, not only the reality of the individual principle considered as if in isolation, but also the “geodetic curvature” of the physical spacetime so defined.

In this sense of the matter, there is a relevant, direct correlation, among: 1) the "cardinality" of typical action within a physical space-time; 2) the order of the Riemannian manifold, which, according to Leibniz' s principle of necessary and sufficient reason, represents that physical space-time; and, 3) the implicitly adducible hypothesis underlying statements expressed in terms of that manifold. It is the correlation of some physical value with the notion of the relative cardinality of the characteristic of action for a given manifold...

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In "non-Euclidean physical geometries," such as that of Riemann's habilitation dissertation, the unit of displacement ("distance") on a curved surface, has an outward appearance which would have been tolerated by Leonhard Euler, et al.; but, underlying that mere appearance, the smallest length of displacement "outwardly" represented by a simple line or arc, is transfinitely dense with "holes," called "discontinuities," sometimes identified by, and sometimes arbitrarily suppressed as, the infinitesimals inhering in the Leibniz calculus . These are each transinfinitesimally small interruptions, which mark the location of an actual, or possible new singularity, such as a new "dimension" of an expanded Riemannian manifold. In other words, we must distinguish between the mere appearance of a simply linear displacement, and the physically efficient content masked by that displacement, the density of discontinuities/singularities.

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“Everything we have said here on this matter, to the present moment of writing, is premised upon, and delimited to statements respecting the set of relations defined by the general principle of hypothesis, even as Riemann’s 1854 habilitation dissertation expresses that Platonic principle as its pivotal foundation. The general set of relations defined by the principle of hypothesis are otherwise describable as relations within an hierarchy of available “pathways of change.” The ordering principle underlying this hierarchy is cardinality, as we have indicated that principle of ordering of Riemannian physical space-time manifolds here. It is in terms of efficient choices of pathways of change, that the future acts upon the present. So, the choice of conception (higher hypothesis) reached with the conclusion of a Classical piece of motivic thorough-composition, determines the potentialities of each subsumed hypothesis, and, thus, of each interval of tolerable counterpoint, within the composition as a whole.”

Lyndon LaRouche, The Essential Role of ‘Time-Reversal’ in Mathematical Economics,

October 11, 1996

First: Any change in an axiomatic assumption, imposed upon us by validated discovery of a revolutionary principle from the domain of experimental physics, establishes a new hypothesis, which supersedes, and is inconsistent with every preceding hypothesis. Each of the two hypotheses , new and old, compared, represents a different manifold, and physical space-time curvature, different from, and functionally inconsistent with the other....

Therefore, the mathematical function containing the transition from phenomena satisfactorily explained by the old hypothesis, to the experimental phenomena characteristic of the new, will be typified by the relevant discontinuity in the function constructed to describe such a case.

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Riemann's mathematical physics requires us to deny primary efficiency to the attributed linear span of displacement, and locate efficiency in the transfinite terms, of density of discontinuities (singularities) per interval of action.

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As I have stressed repeatedly in other location: Knowledge can not be learned; the student must re-create knowledge, by means of reenacting the type of act of discovery experienced, either as by a relevant original discoverer, or based on the model of a subsequent reenactment of that discovery by some relevant person. The act of discovery is not the communication of a literal statement, but, rather, the student's solving of a paradox for which no literal solution is available to him. That solution could not be generated within the bandpass of a medium of communication. That re-discovery may be accomplished, only within the sovereign creative mental processes of each individual person. That process, of evoking a successful reenactment of a discovery of principle, within the sovereign bounds of the individual's cognitive processes, is the only manner in which actual knowledge of a principle could be transmitted. That process of rediscovery (not classroom or textbook learning of successful responses to anticipated multiple-choice questionnaires) is knowledge....

How are singularities, such as metaphors, afforded discrete distinctness within the mind? The answer from any literate person should be: by the juxtaposition which we term irony: a "double meaning," the which can not be resolved deductively.

This may be a valid metaphor, in poetry, Classical drama, painting, or music; or, it may be the introduction of the need to consider a new quality of principle (a new hypothesis), as a precondition for accounting for the actual continuation of a process, as in the case of Riemann's *Fortpflanzung* paper, referenced here earlier.

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Since the history of man 's increase of our species ' power to command the universe to our species ' benefit, is a history of man 's hypothesizing the higher hypothesis, the term "science " is properly- delimited in use to signifying rational comprehension of the process of hypothesizing the higher hypothesis. In that sense, we must think of the subjectivity of science.

Let us agree to name that test of knowledge according to the spirit of Riemann 's experimental physics, "The Great Experiment."

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by Lyndon LaRouche, Oct. 11, 1996**

The lesson of the progress of science, in these, Platonic terms of reference, is that the universe is, in effect, so pre-designed, that it is obliged to obey man's will, whenever man's will is expressed according to Reason: according to valid changes in hypothesis, from lower to higher hypotheses. The relevant action, by means of which the efficient principle of existence of the human species is defined, is the advancement of man's operating hypothesis, from a relatively lower hypothesis, to a relatively more valid, more powerfully efficient one. In effect, the relevant changes are typified mathematically, in the form of an increase of the Gauss-Riemann physical-space-time curvature, by the relative, transfinite cardinality of action.

This is the essence of that which deserves the name of 'science,' or of 'Classical art.'

Lyndon LaRouche, The Essential Role of 'Time-Reversal' in Mathematical Economics

When is the future? At what point in time? Similarly, what is the beginning-point in time from which to define the cumulative past with which the future is to collide? The answer to this seeming paradox, was already known by Plato, by Augustine of Hippo, and, therefore, also, Thomas Aquinas : All time is subsumed under a general regime of simultaneity! The highest expression of change, is that lattice of higher hypotheses which expresses the transfinite notion of hypothesizing the higher hypothesis. What underlies that lattice? That lattice is underlain by what Plato distinguishes as the Good. In the analysis situs of hypothesis, that Good is "simultaneously" efficient in all times and places which might exist. Thus, in those terms of reference, the past and future, as hypothesis, are existent as efficient agency in each present moment.

**The Essential Role of 'Time Reversal' In Mathematical Economics
by Lyndon LaRouche, Oct. 11, 1996**

If I am conscious of the content of my own knowledge and practice, in the manner underlying a Classical humanist form of education, then I know that most of what I know represents valid discoveries of principle effected by individual original discoverers, some known by name, more unknown, most located deep in the lost pages of pre-history. In reenacting their discoveries of principle, I have relived in my mind, moments from the interior of their own. I am closer to these long-deceased persons than to most of the daily associates of my childhood, youth, and adult life. If I aid in transmitting these precious gifts from the past, into the countless generations of the future, and perhaps add one or two such gifts of my own, I am certain that my life will have been a necessary one: both a fulfillment of the past, and a gift to the future. I have thus met 'the test of death.'

Everything we have said here on this matter, to the present moment of writing, is premised upon, and delimited to statements respecting the set of relations defined by the general principle of hypothesis, even as Riemann's 1854 habilitation dissertation expresses that Platonic principle as its pivotal foundation. The general set of relations defined by the principle of hypothesis are otherwise describable as relations within an hierarchy of available "pathways of change." The ordering principle underlying this hierarchy is cardinality, as we have indicated that principle of ordering of Riemannian physical space-time manifolds here.

**The Essential Role of 'Time Reversal' In Mathematical Economics
by Lyndon LaRouche, Oct. 11, 1996**

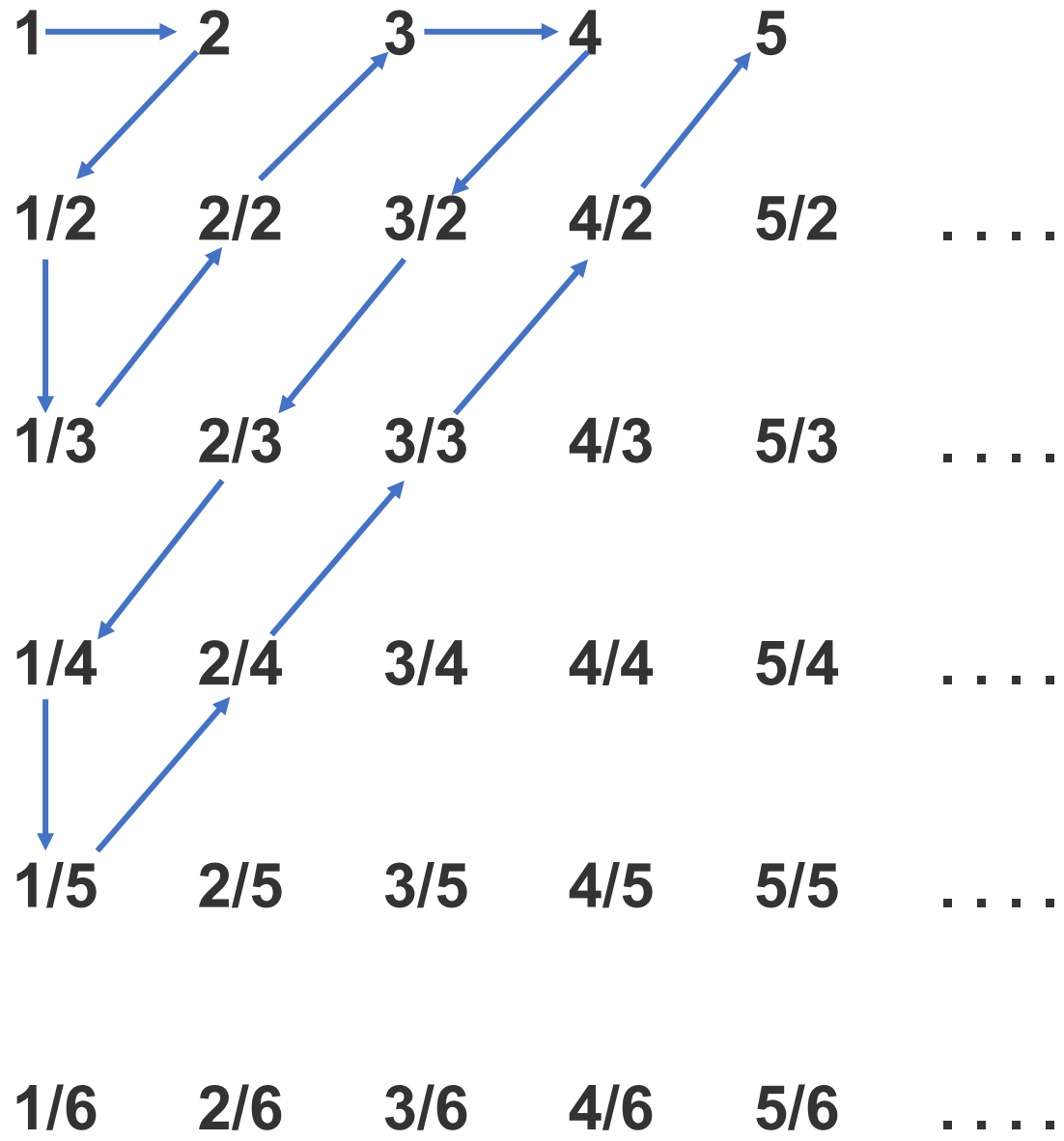
Which Has a Bigger Infinity: Counting Numbers or Even Numbers?

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18

1	2	3	4	5	
1/2	2/2	3/2	4/2	5/2
1/3	2/3	3/3	4/3	5/3
1/4	2/4	3/4	4/4	5/4
1/5	2/5	3/5	4/5	5/5
1/6	2/6	3/6	4/6	5/6

Cantor's Diagonal





Vernadsky, working in the tradition of his predecessor D.I. Mendeleev, and also of the circles of Louis Pasteur, made two, successive great discoveries of universal physical principle, discoveries which divided the domain of physical science and culture among three categorical sets of phenomena, each and all occupying and sharing the same universal physical space-time. These three were: the ordinary space of non-living physical chemistries; the phase-space defined by living processes and their products, called the Biosphere; and, the phase space defined by the products of those processes of the human mind which we should associate with the discovery and use of knowledge of universal physical principles, the Noösphere. Vernadsky defined both the Biosphere and Noösphere as belonging to the domain of a Riemannian manifold, a conclusion which placed Vernadsky in the same domain of intellectual work as his approximate contemporary, the Albert Einstein who traced all ordinary physical chemistry within the domain defined by the line of development of modern physical science, as rooted in the discoveries of Johannes Kepler, and as leading into the discoveries of Bernhard Riemann.

Lyndon H. LaRouche, **"The Rules of Survival"**

May 24, 2007

Although nonliving processes are apparently entropic, relative to the characteristic anti-entropy of living and cognitive processes generally, this does not signify that the non-living aspect of the universe is actually entropic, since, of course, the universe in which the non-living aspect is manifest, is also anti-entropic as a whole, especially when living and cognitive processes are taken into account.

Lyndon H. LaRouche, **“A Lawless USA Today: Faith, Hope, And Agapé!”**
May 13, 2001